

Extra Problems:

EP-1 Happy Birthday to who? ... Alice and Blair are twins, who like to blow out their birthday candles at the same moment each year. Usually, they are together on their birthday. But this year, while Alice is here on earth, Blair is on vacation on a planet many light-years distant.

(a) Assume that Alice and Blair share a single inertial frame, and that their watches have been carefully synchronized. Assume that they agree to blow out candles at noon on September 4, 2001, Eastern Standard (earth) time. They do.

Draw a spacetime diagram from the point of view of an observer halfway between them, and sharing their frame of reference.

Show the following 2 events: Alice blowing out the candles and Blair blowing out the candles.

Show the following 4 worldlines: Alice, Blair, a beam of light from the observer striking Alice just as she blows out the candles, a beam of light striking Blair as he blows out the candles.

(b) Unbeknownst to the twins, a fast rocket is cruising through the galaxy. It is traveling at a constant velocity with respect to the frame of Alice and Blair. It zooms right over Alice's head at the moment she blows out the candles. It is heading directly toward Blair's planet. A clock on board the rocket tells the pilot that on noon of September 24, 2001, Alice blew out her candles.

Does the pilot think they blew out their candles simultaneously? If not, which twin does the pilot think blew out the candles first? Draw another spacetime diagram in order to justify your answer.

EP-2 Two radar pulses sent out from the earth at 6:00 AM and 8:00 AM one day bounce off an alien spaceship and are detected on earth at 3:00 PM and 4:00 PM. You are not sure, however, which reflected pulse corresponds to which emitted pulse, Is the spaceship moving toward earth or away? If its speed is constant (but less than c), when will it (or did it) pass by the earth? Drawing a spacetime diagram will make this problem easy.

EP-3 According to a nuclear treaty between two superpowers, if either strikes first, then the second is entitled to destroy the first completely. The superpowers deploy two ships A and B which move at a very high speed towards each other. Ship A sends off radar signals at one-second intervals which are reflected back by B. At $t=0$ in its coordinates, A fires a weapon at B. At $t=4$, A receives back the signal A sent out at $t=-6$, which detects B firing at A.

What can A conclude about who fired first?

EP-4 Two volcanoes 100 km apart on Io (a satellite of Jupiter) are seen by and observer A at rest on Io to erupt simultaneously. Observer B is the pilot of a rocket, which according to A is 10 km directly above the first volcano when it explodes, flying towards the

second volcano at the speed $1/2$.

What will B see as happening at the second volcano at the moment when she sees the first explode?

EP-5 A rocket R moves in the +x direction relative to an observer A on Mars, at a speed $1/2$. Their positions coincide at $t=0$. Plot the worldlines of A and R in a (t,x) spacetime diagram.

The rocket emits light signals in both the forward and backward directions at $t=2$ sec. Draw the corresponding light rays on your spacetime diagram. The observer A signals to the rocket at time $t=1$ sec. What is the earliest time she can expect to get a reply?

EP-6 Two clocks Imagine two clocks, P and Q. Both clocks leave the spatial origin of the Home Frame at time $t = 0$: call this the origin event O. Both clocks move along the +x axis, with clock P originally traveling at a speed of $4/5$, while Q travels at a speed of about $1/5$. After a while, however, clock P decelerates, comes to rest, and then begins to move back towards the origin. A short time later, clock P collides with the slower clock Q, which has been moving with constant speed along the x axis during all of this. Let the collision of the clocks be event A.

(a) Draw a qualitatively accurate spacetime diagram of the story described above, labeling the worldlines of clocks P and Q and the locations of events O and A.

(b) Assume that clocks P and Q were both synchronized with the clock at the origin of the Home Frame when they left the origin. Will P and Q necessarily agree when they collide? Explain.

(c) An observer in the Home Frame measures the time between O and A with a pair of synchronized clocks (one at the spatial location of O and one at the spatial location of A). Clocks P and Q each also register a time between these events.

Which clock(s) measure proper time between the events? The spacetime interval between the events? The coordinate time between the events?

EP-7 More events in different frames In the solar system frame, two events are measured to occur 3.0 hr apart in time and 1.5 hr apart in space. Observers in an alien spaceship measure the two events to be separated by only 0.5 hr in space. What is the time separation between the events in the alien's frame?

EP-8 Can you save them? In 2095 a message arrives at earth from the growing colony at Tau Ceti (11.3 y from earth). The message asks for help in combating a virus that is making people seriously ill (the message includes a complete description of the virus genome). Using advanced technology available on earth, scientists are quickly able to construct a drug that prevents the virus from reproducing.

You have to decide how much of the drug can be sent to Tau Ceti.

The space probes available on short notice could either boost 200 g of drug (in a standard enclosure) to a speed of 0.95, 1 kg to a speed of 0.90, 5 kg to a speed of 0.80, or 20 kg to a speed of 0.60

relative to the earth.

The only problem is that a sample of the drug in a standard enclosure at rest in the lab is observed to degrade due to internal chemical processes at a rate that will make it useless after 5.0 y.

Is it possible to send the drug to Tau Ceti? If so, how much can you send?

EP-9 They are simultaneous somewhere....

The space and time coordinates of two events as measured in a frame S are as follows:

$$\text{Event 1: } x_1 = x_0, t_1 = x_0/c$$

$$\text{Event 2: } x_2 = 2x_0, t_2 = x_0/2c$$

(a) there exists a frame in which these two events are simultaneous. Find the velocity of that frame(S') with respect to S.

(b) What is the value of t' at which both events occur in the new frame S'?

EP-10 The Strange World of Relativity

Solve this problem with the Lorentz transformation equations and with a spacetime diagram.

At noon a rocketship passes the earth with a velocity of $0.8c$. Observers on the ship and on earth agree that it is noon.

(a) At 12:30 PM as read by a rocketship clock, the ship passes an interplanetary navigational station that is fixed relative to the earth and whose clocks read earth time. What time is it at the station?

(b) How far from earth (in earth coordinates) is the station?

(c) At 12:30 PM rocketship time the ship reports by radio back to earth. When (earth time) does the earth receive the signal?

(d) The station on earth replies immediately. When (by rocket time) is the reply received?

EP-11 Red or Green?

There is a spaceship shuttle service from the earth to Mars. Each spaceship is equipped with two identical lights, one at the front and one at the rear. The spaceships normally travel at a speed v_0 , relative to the earth, such that the headlight of a spaceship approaching the earth appears to be green ($\lambda = 5 \times 10^{-7} m$) and the taillight of a departing spaceship appears to be red ($\lambda = 6 \times 10^{-7} m$).

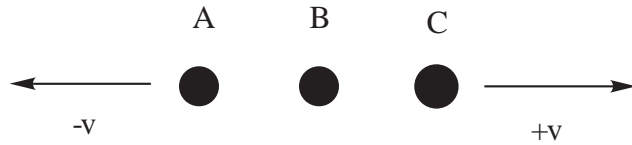
(a) What is the value of v_0 ?

(b) One spaceship accelerates to overtake the spaceship ahead of it.

At what speed must the overtaking spaceship travel (relative to the earth) so that the taillight ($\lambda = 6 \times 10^{-7} m$) of the Mars-bound spaceship ahead of it looks like a headlight ($\lambda = 5 \times 10^{-7} m$)?

EP-12 What is the frequency?

Three identical radio transmitters A, B, and C, each transmitting at the frequency ν_0 in its own rest frame, are in motion as shown below:



- (a) What is the frequency of B's signal as received by C?
- (b) What is the frequency of A's signal as received by C?

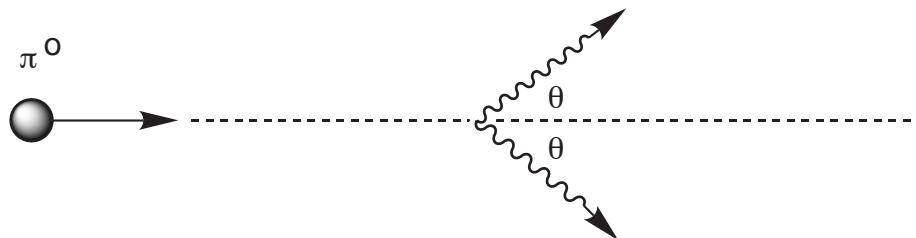
EP-13 In the laboratory frame a particle of rest mass m_0 and a speed v is moving towards a particle of rest mass m_0 that is at rest. What is the speed of the inertial frame in which the total momentum of the system is zero? This frame is called the **center of mass** or **zero momentum** frame.

EP-14 A proton with a kinetic energy of $10^{10} eV$ collides with a proton at rest. Find

- (a) the velocity of the center of mass
- (b) the total momentum and total energy in the laboratory frame
- (c) the kinetic energy of the particles in center of mass frame

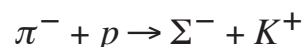
EP-15 Two particles of rest mass m_0 approach each other with equal and opposite velocity v in the laboratory frame. What is the **total energy** of one particle as measured in the rest frame of the other?

EP-16 A π^0 particle of rest mass 135 MeV, **decays** symmetrically (as shown below) into two photons while moving at high speed. The energy of each photon in the laboratory system is 100 MeV.



- (a) Find the meson's speed v .
- (b) Find the angle θ in the laboratory between the momentum of each photon and the initial line of motion of the meson.

EP-17 Determine the **threshold energy** for the reaction



Assume that the negative pi-meson is the incident particle and the proton is initially at rest.