

EPR and Bell Locality

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A new formulation of the EPR argument is presented, one which uses John Bell's mathematically precise local causality condition in place of the looser locality assumption which was used in the original EPR paper and on which Niels Bohr seems to have based his objection to the EPR argument. The new formulation of EPR bears a striking resemblance to Bell's derivation of his famous inequalities. The relation between these two arguments – in particular, the role of EPR as part one of Bell's two-part argument for nonlocality – is also discussed in detail.

I. INTRODUCTION

Eugene P. Wigner summed up what has since become the standard view of the implications of Bell's Theorem¹ when he stated: "In my opinion, the most convincing argument against the theory of hidden variables was presented by J. S. Bell.The...argument shows that any theory of hidden variables conforming to the postulate of locality is in conflict with quantum mechanics."²

N. David Mermin echoed this view in his characteristically brilliant review article on "Hidden Variables and the Two Theorems of John Bell":

Bell's theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained – a fact that Bohm theory exemplifies, and that is now understood to be an unavoidable feature of any hidden-variables theory.

To those for whom nonlocality is anathema, Bell's Theorem finally spells the death of the hidden-variables program.³

This, then, is the standard view: Bell proved that hidden-variables theories have to be nonlocal (in order to agree with the empirically correct predictions of quantum mechanics); nonlocality conflicts with relativity's prohibition on super-luminal causation; relativity is true; so hidden-variables theories must be false.

Yet somehow this obvious and now standard argument seems to have escaped Bell himself – who, according to Mermin's summary, "did not believe that either of his no-hidden-variables theorems excluded the possibility of a deeper level of description than quantum mechanics."⁴ How strange! *Bell himself did not believe* that what Mermin refers to as Bell's two *no-hidden-variables theorems*, actually exclude hidden-variables! Why didn't Bell accede to the standard view of his own theorems? Was this universally-recognized genius really so obtuse?

Mermin provides a clue in the continuation of the above block-quote:

But not for Bell. None of the no-hidden-variables theorems persuaded him that hidden-variables were impossible. What Bell's Theorem did suggest to Bell was the need to reexamine our understanding of Lorentz invariance...⁵

Thus Bell believed that his theorems brought out a conflict not merely between relativity and hidden-variables theories, but, rather, between relativity and the predictions of quantum theory *as such*, in any interpretation. Mermin briefly mentions this possible view in a footnote: "Many people contend that Bell's Theorem demonstrates nonlocality independent of a hidden-variables program, but there is not general agreement about this."⁶

Evidently the "many people" referred to here by Mermin include in their ranks Bell himself.

Given the unique clarity and forthrightness of Bell's writings, it is not surprising that we needn't undertake extensive detective work to infer Bell's views. He tells us quite explicitly both *that* and *why* he believes his theorems call into question our understanding of fundamental space-time structure, and not merely the attempt to supplement quantum mechanics with additional variables.

Here is the *that*: "...the nonlocality of quantum mechanics cannot be attributed to incompleteness, but is somehow irreducible."⁷ Also: "The obvious definition of 'local causality' does not work in quantum mechanics, and this cannot be attributed to the 'incompleteness' of that theory."⁸ And: "For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory..."⁹

And here is the *why*: "That ordinary quantum mechanics is not locally causal was pointed out by Einstein, Podolsky, and Rosen, in 1935."¹⁰ That is, according to Bell, the reason Bell's Theorem spells trouble (in the sense of a conflict with relativity) for more than just hidden-variables theories is that standard quantum mechanics itself (regarded as an already-complete description of the world) is nonlocal – a fact which Bell claims was pointed out in the famous 1935 EPR paper.

But this raises another mystery. Everyone knows that the purpose of the EPR paper was not to argue that quantum mechanics (QM) is nonlocal, but, rather, to argue against the completeness doctrine. Indeed, the EPR argument was crucially premised on the very assumption – locality – that Bell claims EPR disproved: according to EPR, if

(A) the quantum-mechanical predictions for certain testable experiments are correct

and if

(B) quantum theory is taken to be a locally causal theory

then the claim that

(C) the theory is complete

cannot be correct. Or, taking as indubitable the (experimentally well-confirmed) empirical predictions of quantum theory referred to in (A), the EPR argument takes the form:

$$(B) \rightarrow \neg(C). \quad (1)$$

That is, locality implies *incompleteness*. (We use the symbol \neg to denote negation: e.g., $\neg(X)$ should be read “it is not the case that (X)”.)

But this is logically equivalent to the claim that

$$(C) \rightarrow \neg(B) \quad (2)$$

(i.e., completeness implies nonlocality) and also to the claim

$$\neg(B) \text{ or } \neg(C). \quad (3)$$

(i.e., either locality or completeness must fail). Einstein himself stated the conclusion of the EPR argument in this last form:

By this way of looking at the matter, it becomes evident that the paradox [EPR] forces us to relinquish one of the following two assertions:

(1) the description by means of the ψ -function is complete.

(2) the real states of spatially separated objects are independent of each other.¹¹

Evidently, then, this is the basis for Bell’s assertion that EPR showed that “ordinary quantum mechanics is not locally causal.” For if we grant the premise (surely ordinary, ever since the 1930’s at least) that QM is *complete*, it follows from the EPR argument that ordinary QM itself is *nonlocal*. So if the EPR argument is sound – if it is correct that quantum mechanics, if complete, is nonlocal – then Bell’s own interpretation of the significance of his theorems (and not the more widely-held interpretation put forward by Wigner and Mermin) would be validated.

But is the EPR argument sound? The standard view in the physics community has been that Niels Bohr refuted the EPR argument in 1935 by pointing out an “essential ambiguity” in the famous EPR criterion of reality: “If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”¹²

Bohr claimed that “the wording of the above mentioned criterion...contains an ambiguity as regards the meaning of the expression ‘without in any way disturbing a system’.” That is, Bohr seems to have objected to the formulation of *locality* which entered into the EPR reality criterion. In particular, he argued that there was a type of non-mechanical disturbance which EPR had neglected and that the criterion was therefore inapplicable to the very example on which they base their argument: “Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an *influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term ‘physical reality’ can be properly attached, we see that the argumentation of [EPR] does not justify their conclusion that [, if local, the] quantum-mechanical description is essentially incomplete.”¹³

Many commentators (including, not surprisingly, Bell) have questioned the validity, clarity, and relevance of Bohr’s reply.¹⁴ I agree with Bell and the other critics that the EPR argument as originally formulated is perfectly sound and that Bohr’s reply in no way refutes it. Nevertheless, it is true that the exact definition of locality used as a crucial premise in the EPR argument – and also the exact role of that premise in the argument – are less than crystal clear. It would be desirable, therefore, if the concept of local causality could be clarified, and the EPR argument reformulated in terms of this clearer concept. This is the goal of the present paper.

Happily, there is almost no work to do to achieve this goal – for in the course of establishing his “no-hidden-variables” theorems, John Bell introduced an intuitive and mathematically precise definition of local causality. So the goal at hand can be achieved simply by replacing EPR’s somewhat vague language about not disturbing a distant system with the quantitative requirement of Bell Locality. We will perform this replacement in Section III after first, in Section II, briefly reviewing the original EPR argument. Finally, in Section IV, we discuss the relation of the re-formulated EPR argument to Bell’s Theorem – in particular, the role of the EPR argument in Bell’s two-part argument for nonlocality.

II. THE EPR ARGUMENT

Before presenting the updated version, let us briefly recap the original EPR argument. We will use the simplified example introduced by Bohm¹⁵ in which different spin components of two spin-1/2 particles take the place of the position and momentum variables used in the original EPR paper. The two versions, however, are identical in terms of logical structure, so we will refer freely to the original EPR paper as if they had based the argument on Bohm's example. Our goal in this section is simply to lay out the logical structure of the EPR argument, so that we can provide a recognizably similar structure in the next section (but with Bell Locality in place of EPR's looser locality assumption).

Consider two spin-1/2 particles which are spatially separated but in the spin singlet state:

$$\psi_0 = \frac{1}{\sqrt{2}} (|+z\rangle_1 |-z\rangle_2 - |-z\rangle_1 |+z\rangle_2) \quad (4)$$

where $|+z\rangle_1$ is the state in which particle 1 has spin + along the z-axis, etc. Equation 4 attributes no definite spin values to either of the two particles separately, but it does imply a definite relation between the spins: whatever the spin of particle 1 is (measured to be) along the z-axis, the spin of particle 2 along the z-axis will be (measured to be) opposite.

The same singlet state can also be written in other bases, e.g., the basis of eigenstates for spin along the x-axis:

$$\psi_0 = \frac{1}{\sqrt{2}} (|+x\rangle_1 |-x\rangle_2 - |-x\rangle_1 |+x\rangle_2). \quad (5)$$

So in addition to the perfect (anti-)correlation of (measured) spin values along the z-axis mentioned above, there is also a perfect (anti-)correlation of (measured) spin values along the x-axis: if we measure the spin of particle 1 along the x-axis and find the result +, we can be certain that the spin of particle 2 along the x-axis (if/when measured) will be -, and vice versa.

The just-described correlations represent the predictions of quantum theory for this sort of situation. But it is more important here that these correlations are empirically well-verified, i.e., that the claimed perfect anti-correlation of spin values (when the spin of both particles is measured along the same axis) is *true*.¹⁶ Whichever way one prefers to motivate it, however, this perfect anti-correlation is the first assumption of the EPR argument. We will refer to this group of assumptions as "EPR(A)".

The EPR argument then proceeds as follows.

According to the EPR criterion of reality, "If, without in any way disturbing a system, we can predict with certainty ... the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." Let us simply assume that, since particles 1 and 2 are spatially separated, the act of measuring the spin of particle 1 along the z-axis doesn't disturb

particle 2 in any way, and likewise for a measurement of the spin of particle 1 along the x-axis. (This assumption is obviously motivated by relativity's prohibition on causal relations between space-like separated events – a relationship that the two measurement events in question here can simply be stipulated to have.) Let us call this locality assumption "EPR(B)".

EPR now argue: by measuring the z-axis spin of particle 1, we can determine the z-axis spin of particle 2 by using EPR(A) and, of course, without in any way disturbing particle 2. There exists, therefore, an element of reality corresponding to the z-axis spin of particle 2. Why? Because after the measurement on particle 1, particle 2 is known to be in a state with a definite value of spin along the z-axis. This follows from a trivial application of EPR(A). But, by EPR(B), the measurement on particle 1 could not have caused particle 2 to acquire this property, for particle 2 was not disturbed in any way by the measurement on 1. Thus if particle 2 has this property after the measurement, it must evidently have possessed this property all along, independent of the measurement made on particle 1. The measurement on 1, if/when performed, permits us to *learn* something about the z-axis spin of particle 2. But the fact we learn about *exists* (i.e., is an element of reality) independent of that measurement.

The same argument obviously goes through for the x-axis spin as well: by measuring the x-axis spin of particle 1, we can determine without in any way disturbing particle 2, the x-axis spin of particle 2. There exists, therefore, an element of reality corresponding to this property of particle 2 as well.

The reader might perhaps worry that either one or the other of these arguments can be validly made in a given experimental situation, but both cannot be, since we can measure at most one of the two relevant properties of particle 1 (and hence infer using EPR(A) only one of the two relevant properties of particle 2). EPR answer this possible worry in their paper:

One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. On this point of view, since either one or the other, but not both simultaneously, of the quantities ... can be predicted, they are not simultaneously real. This makes the reality of [the two properties of particle 2] depend upon the process of measurement carried out on [particle 1], which does not disturb the second [particle] in any way. No reasonable [i.e., local] definition of reality could be expected to permit this.¹⁷

Thus – although the operators for these two observables

don't commute and therefore cannot according to quantum mechanics possess simultaneous definite values – the x-axis and z-axis spins of particle 2 *do* possess simultaneous definite values. There are elements of reality corresponding to both quantities. And that means the descriptive limitations imposed by QM (expressed most pointedly by the Heisenberg Uncertainty Principle) can be beaten: there are more facts out there in the world than can be squeezed into the quantum mechanical description.¹⁸ We thus arrive at the negation of “EPR(C)” – the claim that the quantum mechanical description of reality can be considered complete.

The EPR argument thus takes the symbolic form:

$$\text{EPR(A) and EPR(B)} \rightarrow \neg \text{EPR(C)} \quad (6)$$

which is logically equivalent to

$$\text{EPR(A) and EPR(C)} \rightarrow \neg \text{EPR(B)} \quad (7)$$

and also to

$$\neg \text{EPR(A) or } \neg \text{EPR(B) or } \neg \text{EPR(C)} \quad (8)$$

which is Einstein's formulation quoted above: (given the empirically well-verified quantum mechanical expressions for certain correlations) we are “forced to relinquish” either locality [EPR(B)] or completeness [EPR(C)].

III. THE EPR-BELL ARGUMENT

Like the EPR argument just considered, the new version of EPR (let us call it the EPR-Bell argument, since it is based so heavily on the reasoning of Bell) begins with Bohm's example of a system consisting of two spatially separated spin-1/2 particles in the spin-singlet state, Equation 4.

Let us first introduce the analog of EPR(A) for the EPR-Bell argument. To make the argument as straightforward as possible, we will use here a slightly-expanded set of empirical predictions compared to the simple perfect anti-correlation used in the original EPR argument. But, like the assumption EPR(A) above, these predictions will all be straightforward, uncontroversial predictions of QM that are well-confirmed by experiment.

First, we introduce the probability for joint outcomes (A and B) for spin measurements along arbitrary directions \hat{a} and \hat{b} on the two particles (respectively) in the singlet state ψ_0 of Equation 4:

$$P(A=+, B=+ | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \sin^2(\theta/2) \quad (9)$$

$$P(A=+, B=- | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \cos^2(\theta/2) \quad (10)$$

$$P(A=-, B=+ | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \cos^2(\theta/2) \quad (11)$$

$$P(A=-, B=- | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \sin^2(\theta/2). \quad (12)$$

where θ is the angle between \hat{a} and \hat{b} . Call this set of assertions “EPR-Bell(A1)”.

We also note the standard quantum expressions for the marginal probabilities for the outcomes of spin measurements on each particle individually:

$$P(A=+ | \hat{a}, \psi_0) = 1/2 \quad (13)$$

$$P(A=- | \hat{a}, \psi_0) = 1/2 \quad (14)$$

$$P(B=+ | \hat{b}, \psi_0) = 1/2 \quad (15)$$

$$P(B=- | \hat{b}, \psi_0) = 1/2. \quad (16)$$

This set of expressions – “EPR-Bell(A2)” – simply states that with the particles in the state ψ_0 , we are equally likely to get a + or – outcome for any single measurement on a single particle, independent of the angles \hat{a} and \hat{b} .

With that set of assumptions on the table, let us proceed with the argument.

Consider now a general expression for the joint probability for the two outcomes A and B , when the spin values along directions \hat{a} and \hat{b} , respectively, are measured:

$$P(A, B | \hat{a}, \hat{b}, \lambda). \quad (17)$$

Here λ is a complete specification of the physical state of the particle pair prior to measurement.

Let us introduce now “EPR-Bell(B)” – the requirement of Bell Locality – according to which the joint probability $P(A, B | \hat{a}, \hat{b}, \lambda)$ should *factor* into a product of individual probabilities for the two spatially separated systems:

$$\begin{aligned} P(A, B | \hat{a}, \hat{b}, \lambda) &= P(A | B, \hat{a}, \hat{b}, \lambda) \times P(B | \hat{a}, \hat{b}, \lambda) \\ &= P(A | B, \hat{a}, \lambda) \times P(B | \hat{b}, \lambda) \\ &= P(A | \hat{a}, \lambda) \times P(B | \hat{b}, \lambda). \end{aligned} \quad (18)$$

We have moved in small steps here to make clear several sub-aspects of Bell locality.¹⁹

The equality in the first line is standard conditional probability, and should be completely uncontroversial.

The move from here to the second line involves an application of what Abner Shimony has dubbed “Parameter Independence” (PI).²⁰ This principle asserts that the probabilities associated with particular outcomes for each particle should be independent of which property is measured on the distant particle – e.g., $P(B | \hat{a}, \hat{b}, \lambda) = P(B | \hat{b}, \lambda)$. The physical intuition motivating PI is that any such stochastic dependence could only be accounted for by a “spooky nonlocal action at a distance” by which the setting of the distant instrument somehow causally influenced the (probability distribution of) results of the nearby experiment.

Finally, the move to the third line utilizes what Shimony calls “Outcome Independence” (OI), according to which the probabilities associated with particular outcomes for each particle should be independent of the outcome (+ or -) of the distant experiment. Again, this seems to be an aspect of the more general (locality) requirement that what happens *here* should be independent of what happens *over there* (or, more precisely, that

correlations between space-like separated events can be accounted for by facts in the union of the past light-cones of the two detection events – here, by the pre-existing, complete, joint state of the two particles: λ).

(It should also be noted that Jon Jarrett²¹ was the first to point out that Bell Locality was entailed by the conjunction of the two principles PI and OI, though Jarrett referred to these principles by different names. Tim Maudlin, however, has quite reasonably criticized Jarrett’s analysis of Bell’s factorization principle.²² As Maudlin points out, Jarrett’s parsing is not unique, so the relevance of the distinction between PI and OI is called into question. Moreover, Jarrett originally argued that a failure of PI would mean a violation of relativity, while a failure of OI – which, incidentally, is the particular aspect of Bell Locality violated by standard QM – would not. But, as Maudlin makes clear, this makes no sense: if relativity is taken to prohibit causal dependency between space-like separated events, violations of PI and OI are equally at odds with it. The reader is urged to consult Maudlin’s excellent text for a much more extensive and highly enlightening discussion.)

In any case, what concerns us here is simply that the condition expressed by Equation 18 – “EPR-Bell(B)” – is the one typically used in deriving Bell’s Theorem. Our goal here is not primarily to argue for the validity or appropriateness of this condition (though for the record we believe it is both entirely valid and entirely appropriate) but merely to show that the condition can be used to reformulate the EPR argument. Let us therefore proceed with that argument.

The third principle relevant to our derivation is the completeness assumption. We have previously introduced the symbol λ to refer to a complete specification of the joint pre-measurement two-particle state. In typical derivations of Bell’s theorem, this symbol refers to the wave function *plus* whatever hidden-variables are needed to complete the (in that context, assumed incomplete) quantum description. But we are not here reproducing Bell’s theorem. We are instead aiming to reproduce the EPR argument, so we will assume with Bohr that *quantum mechanics itself is already complete, without the addition of any hidden-variables*. We will then demonstrate that there is a contradiction implied by the four assumed principles, and hence arrive at the EPR conclusion that either locality or completeness – or, far less plausibly, one of EPR-Bell(A1) or EPR-Bell(A2) – must fail.

Thus, let us now formally make the completeness assumption – i.e., replace λ with the appropriate quantum mechanical wave function, ψ_0 . Call this replacement “EPR-Bell(C)”.

We may now combine EPR-Bell(B) with EPR-Bell(C). The result is:

$$P(A, B | \hat{a}, \hat{b}, \psi_0) = P(A | \hat{a}, \psi_0) \times P(B | \hat{b}, \psi_0). \quad (19)$$

This leaves us in a position to utilize the expressions in EPR-Bell(A2). Plugging in yields:

$$P(A=+, B=+ | \hat{a}, \hat{b}, \psi_0) = 1/4 \quad (20)$$

and

$$P(A=+, B=- | \hat{a}, \hat{b}, \psi_0) = 1/4 \quad (21)$$

and

$$P(A=-, B=+ | \hat{a}, \hat{b}, \psi_0) = 1/4 \quad (22)$$

and

$$P(A=-, B=- | \hat{a}, \hat{b}, \psi_0) = 1/4. \quad (23)$$

These expressions have been deduced by straightforwardly combining EPR-Bell(A2), (B), and (C) – i.e., the quantum expressions for marginal probabilities, Bell Locality, and the assumption that the quantum mechanical description of physical reality is complete. And, as should be obvious, these predictions conflict with the expressions in EPR-Bell(A1). We have thus proved that all four of these principles cannot be simultaneously correct. At least one member of the set must be false.

And since EPR-Bell(A1) and EPR-Bell(A2) are both directly supported by experiment, we must evidently reject either EPR-Bell(B) – (Bell) Locality – or EPR-Bell(C) – completeness – in order to avoid this inconsistency. This of course matches the conclusion reached by EPR.

IV. DISCUSSION

We have shown that it is possible to reformulate the EPR argument by using Bell’s mathematically precise local causality requirement, and that doing so permits the EPR argument to go through as intended by its authors.

It is suspected that some readers might initially reject the argument presented in Section III on the grounds that it does not fairly represent quantum mechanics. For example, Equation 18 requires that the joint probability for outcomes of the spin measurements on particles 1 and 2 *factor*, but (as mentioned in Section II) quantum mechanics attributes *no definite spin states* to the individual particles. Instead, QM attributes a definite (spin) state (namely ψ_0) only to the whole two-particle system. So *of course*, the skeptic will say, the mathematical consequences of this factorizability assumption will not be consistent with quantum mechanics and will not be consistent with experiment.

Unfortunately for this imagined skeptic, however, this objection does nothing to undermine the validity of the *argument* presented in Section III; it merely denies one of the *premises* of that (reductio) argument – namely, the Bell Locality requirement, EPR-Bell(B). But surely the denial of EPR-Bell(B) is entirely consistent with the conclusion of the argument, namely that at least one of the (four) premises must be false. If one wishes to deny that conclusion, one must identify an error in the reasoning (and not simply assert something that is consistent with the conclusion).

Lest the reader worry that I am setting up and knocking down a straw man by proposing this rather silly attempt at an objection, let me point out that the imagined objection is highly reminiscent of Bohr’s objection to the original EPR paper:

There is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term ‘physical reality’ can be properly attached, we see that the argumentation of [EPR] does not justify their conclusion that quantum-mechanical description is essentially incomplete.²³

Bohr’s first sentence appears to indicate agreement with the locality principle, EPR(B). Bohr concedes that particle 2 is not “mechanically” disturbed by the measurement on particle 1. Yet he goes on to say that there is some other kind of disturbance – “an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.” It is not entirely clear what this is supposed to have meant. If this influence is (while not “mechanical”, still) a physical disturbance – if it in any way changes the physical state of particle 2 – then Bohr’s answer simply concedes that quantum theory is nonlocal. Bell speculated that Bohr’s answer was indeed this same silly objection I mentioned above: “Is Bohr just rejecting the premise – ‘no action at a distance’ – rather than refuting the argument?”²⁴

If, on the other hand, this “disturbance” isn’t physical but merely “semantic”²⁵ – if we are merely learning something about particle 2 which was before unknown – then Bohr’s answer simply concedes that quantum theory is incomplete.

Thus, in response to the EPR argument which concluded that either EPR(B) or EPR(C) must be false, Bohr seems to have replied in a way that can be interpreted as meaning either that EPR(B) is false, or that EPR(C) is false. Presumably Bohr didn’t intend either of these clear-cut options, since he seemed to think that he had refuted the EPR argument. But no alternative options present themselves. Unless some other alternative is found and made clear, we must evidently conclude that the EPR argument is indeed sound. Quantum mechanics cannot be regarded as simultaneously (A) yielding correct predictions, (B) respecting the principle of local causality, and (C) providing a complete description of physical reality.

One final point on possible objections to the argument presented in Section III: as will be discussed in some detail shortly, the argument there mirrors in several cru-

cial respects Bell’s derivation of his famous inequalities. So (virtually) any objection one might possibly raise to the reformulated EPR argument would simultaneously constitute an objection to Bell’s theorem. Opponents of the hidden-variables program tend to side with Bohr in dismissing the EPR argument (considered as an argument against the completeness doctrine), and simultaneously to regard Bell’s Theorem as a valid proof of the non-viability of hidden-variables theories. (Indeed, this is merely the conjunction of the two “standard views” mentioned in Section I.) It is hoped that the reader will begin to see the inconsistency of these two views in light of the discussion to which we now turn.

What exactly, then, is the relation between the argument in Section III and the arguments leading to Bell’s Theorem? To begin with, the overall structure of the two arguments is similar: some empirically well-tested facts²⁶ plus the Bell Locality condition plus a completeness assumption²⁷ together yield a prediction which is inconsistent with another empirically well-tested fact²⁸.

But more importantly, the real *meat* of both derivations – the imposition of Bell Locality – is of course completely identical in the two cases. So in a way, our reformulation of EPR can be seen as simply an *application* of Bell’s reasoning to the particular case of QM: we have, at the appropriate juncture, simply replaced Bell’s symbol λ with the quantum state ψ_0 .

Although the two arguments are similar, however, they are certainly not *equivalent*. The (reformulated) EPR argument is an argument *about quantum mechanics*. It says that if quantum mechanics provides a complete description of physical reality (and if its predictions are correct), then the theory violates Bell Locality. Bell’s Theorem yields a similar conclusion, but *about hidden-variables theories*: if we try to save local causality for quantum theory by introducing hidden-variables, the resulting theory (again assuming the predictions of quantum mechanics are correct, which they seem to be) will also have to be nonlocal.

Having mentioned above the similarity between our reformulated EPR argument and Bell’s theorem, it is worth pointing out exactly how and where the two arguments diverge. Most notably, after paralleling Bell and imposing the local causality condition, the EPR-Bell argument from Section III simply ends – the additional processing required to derive Bell’s Theorem is simply not needed. Let us analyze this in detail.

Bell, after arguing for the appropriateness of Equation 18, defines the correlation function

$$E(\hat{a}, \hat{b}) = \sum_{\lambda} \sum_{A, B} A B P(A|\hat{a}, \lambda) P(B|\hat{b}, \lambda) P(\lambda) \quad (24)$$

from which it is simple to arrive at, e.g., the Clauser-Holt-Horne-Shimony Inequality constraining the correlations for different pairs of angles:

$$|E(\hat{a}_1, \hat{b}_1) - E(\hat{a}_1, \hat{b}_2)| + |E(\hat{a}_2, \hat{b}_1) + E(\hat{a}_2, \hat{b}_2)| < 2 \quad (25)$$

Bell then points out that “according to quantum mechanics, this expression can approach $2\sqrt{2}$. So quantum mechanics *cannot* be embedded in a locally causal theory.”²⁹

For our purposes, however, it was sufficient merely to derive Equations 20-23 and point out the inconsistency with EPR-Bell(A1). The EPR conclusion was easier to arrive at because we were working with a definite theory – namely, quantum mechanics – with its explicit candidate for the complete description λ and its explicit expressions for the probabilities in EPR-Bell(A1) and (A2). The hard work in arriving at Bell’s inequalities is in finding a way to constrain the correlations when the theory under scrutiny is left largely unspecified. This hard work – the arguments leading from where we left off to Equation 25 – was achieved brilliantly by Bell and the subsequent researchers (such as Clauser, Holt, Horne, and Shimony) who have discovered additional constraints. Compared to the difficulty of constraining the correlations that can be explained by local hidden-variables theories, then, ours was an almost embarrassingly easy task. It is quite trivial to bring out the inconsistency of the quantum mechanical predictions – EPR-Bell(A1) and EPR-Bell(A2) – when the theory is restricted by both the completeness and Bell Locality assumptions.

The ease of deriving inconsistencies for the two theories reflects the strength of correlations each theory permits. The combination of correlation functions appearing on the left hand side of Equation 25 cannot exceed 2 in a general local hidden-variables theory. But as can be seen from our analysis in Section III, standard quantum theory (assumed complete and Bell-Local and using the predictions from Equations 13-16) predicts *no correlations at all*: the correlation coefficient $E(\hat{a}, \hat{b})$ one gets by plugging Equations 20-23 into Equation 24 is *zero*! And so, therefore, is that combination of correlation coefficients appearing in Equation 25.

That is, if we rigidly impose the (Bell) Locality and completeness assumptions on quantum theory, it becomes a much poorer theory (judged by its empirical predictions) than local hidden-variables theories: the latter come closer to explaining the empirically observed correlations than the former (though both are inconsistent with the data).

This makes perfect sense. Bell’s inequality puts a limit on the strength of correlations between the outcomes of distant measurements which can be explained by a local hidden-variables theory. But what is quantum mechanics but a hidden-variables theory unencumbered by any hidden-variables? Surely it stands to reason that a local theory *with* hidden variables should be able to explain stronger correlations than one without those variables – for the latter is merely an especially wimpy version of the former. So it should not be surprising at all that in a fair fight – i.e., with the same locality condition applied to the two theories (one containing the wave function plus an open-ended slew of local hidden variables, the other containing just the wave function) – the latter should generally exhibit far weaker correlations than the former.

Opponents of hidden-variables theories like to point out that such theories are ruled out by experiment, and often quote the number of standard deviations by which the predictions of local hidden-variables theories disagree with the most recent experiments. It should now be clear that any experiment which measures a violation of Inequality 25 (and thus rules out local hidden-variables theories) also provides the necessary empirical input to demonstrate the inconsistency arrived at in Section III. That is, the data from any experimental violation of Bell’s inequalities is simultaneously proof of the validity of the (reformulated) EPR argument – i.e., proof that quantum mechanics itself is either incomplete or nonlocal.

Let us briefly summarize the implications of this. (We will from now on assume the validity of EPR(A), EPR-Bell(A1), and EPR-Bell(A2).)

The EPR argument tells us that *if quantum mechanics is complete, Bell Locality must fail*:

$$\text{EPR: Completeness} \rightarrow \neg \text{Bell Locality.} \quad (26)$$

Bell’s Theorem tells us that *if quantum mechanics is incomplete, Bell Locality must fail*:

$$\text{Bell: Incompleteness} \rightarrow \neg \text{Bell Locality.} \quad (27)$$

Putting these two arguments together forces us to conclude (without qualification, for surely QM either is or is not complete) that Bell Locality fails:

$$\text{EPR} + \text{Bell} : \neg \text{Bell Locality.} \quad (28)$$

This explains why Bell himself understood his theorem not as simply ruling out the hidden-variables program, but rather as evidencing a deep conflict between quantum theory as such, in any interpretation, and the locality principle that seems to be a requirement of relativity. This seems to have been misunderstood largely because it has not been grasped that Bell’s Theorem is the *second part* of a *two-part argument* for the conclusion.³⁰ The necessary first part of that argument is nothing but EPR, which generations of physicists have claimed was refuted by Bohr. But, simply put, it wasn’t – as the new formulation presented in Section III should help make clear.

As discussed in the introduction, it is commonplace to refer to the famous theorems of John Bell as “no hidden-variables theorems”. But, considering the above, it would be much more accurate to call them “nonlocality theorems”: given that the EPR argument is sound, what Bell’s theorem proves is that the predictions of quantum mechanics for certain experimental results – predictions that have stood up to the test of experiment – are inconsistent with the principle of local causality, period. In particular, as Bell himself stressed, this conclusion – the failure of local causality for QM – “cannot be attributed to the ‘incompleteness’ of that theory.”³¹

Or, as Bell himself elaborated the complete *two-part argument* for nonlocality:

Let me summarize once again the logic that leads to the impasse. The EPRB [i.e., EPR-Bohm – the EPR argument using Bohm’s example] correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. [That is the EPR argument – part 1 of Bell’s 2-part argument.] But this has implica-

tions for non-parallel settings which conflict with those of quantum mechanics. [That is Bell’s Theorem – part 2.] So we *cannot* dismiss intervention on one side as a causal influence on the other. [...regardless of whether or not hidden-variables are added to quantum theory!]³²

It is hoped that the current paper will begin to overturn a truly unfortunate historical injustice – namely, the idea (implied by the standard, almost universally-held interpretation of the meaning of Bell’s Theorem) that John Bell failed to understand his own most important insight.

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- ¹ John S. Bell, *Speakable and Unsayable in Quantum Mechanics*, 2nd ed., Cambridge University Press, 2004
- ² Eugene P. Wigner, “Interpretation of Quantum Mechanics”, 1976, reprinted in “Quantum Theory and Measurement”, John A. Wheeler and Wojciech H. Zurek, editors, Princeton University Press, 1983, pages 260-314
- ³ N. David Mermin, “Hidden Variables and the Two Theorems of John Bell”, *Rev. Mod. Phys.*, Vol. 65, No. 3, July 1993, pages 803-815.
- ⁴ *Ibid.*
- ⁵ *Ibid.*
- ⁶ *Ibid.*
- ⁷ Bell, op cit, pg 244
- ⁸ *Ibid.*, pg 256
- ⁹ *Ibid.*, pg 172
- ¹⁰ *Ibid.*, pg 24
- ¹¹ Albert Einstein, from the “Reply to Criticisms” essay in “Albert Einstein: Philosopher Scientist”, P.A. Schilpp, ed., Harper and Row, 1959, pg 681.
- ¹² Albert Einstein, Boris Podolsky, and Nathan Rosen, “Can Quantum-Mechanical Description of Physical Reality be Considered Complete?”, *Phys. Rev.* 47, 777-80 (1935). Also reprinted in Wheeler and Zurek, op cit.
- ¹³ Niels Bohr, “Can Quantum-Mechanical Description of Physical Reality be Considered Complete?”, *Phys Rev*, 48, 696-702; reprinted in Wheeler and Zurek, op cit. Emphasis in original.
- ¹⁴ See in particular pages 155-6 of Bell, op cit, for his lucid analysis of Bohr’s reply to EPR.
- ¹⁵ David Bohm, “Quantum Theory”, Prentice-Hall, Inc., 1951, pages 611-23.
- ¹⁶ More precisely, it is the optical analogue of these correlations which have been directly tested in experiments.
- ¹⁷ Einstein, Podolsky, and Rosen, op cit.
- ¹⁸ Although the conclusion of incompleteness in the EPR paper was framed around the notion of beating the uncertainty principle, this seems not to have been the argument that Einstein himself favored. See T. Norsen, “Einstein’s Boxes”, quant-ph/0404016 and AmJPhys forthcoming, and references therein.
- ¹⁹ See Bell’s lengthier and clearer discussion of this result in the essay “La Nouvelle Cuisine” in Bell, op cit.
- ²⁰ Abner Shimony, “Our Worldview and Microphysics” in “Philosophical Consequences of Quantum Theory”, James T. Cushing and Ernan McMullin, editors, University of Notre Dame Press, 1989
- ²¹ Jon Jarrett, “On the Physical Significance of the Locality Conditions in the Bell Arguments”, *Nous*, 18, 569-89 (1984)
- ²² Tim Maudlin, *Quantum Non-Locality and Relativity*, 2nd ed., Blackwell Publishing, 2002, Chapter 4.
- ²³ Niels Bohr, op cit
- ²⁴ Bell, op cit, pg 156
- ²⁵ An amusing bit of terminology suggested by Arthur Fine in “The Shaky Game”, The University of Chicago Press, 1996, pg 35.
- ²⁶ namely, EPR-Bell(A2) in our argument, and the perfect anti-correlation of outcomes when measurements are made with $\hat{a} = \hat{b}$ in Bell’s argument
- ²⁷ which, in Bell’s argument, is hardly any assumption at all since the “complete description” λ is never *specified*
- ²⁸ that is, leaving aside detector efficiency issues, in the case of Bell
- ²⁹ Bell, op cit, 244
- ³⁰ The idea and phraseology of a “two-part argument” is due to Prof. Sheldon Goldstein. I was motivated to write the current essay after struggling to come to grips with the arguments outlined in Section II of Goldstein’s article on Bohmian Mechanics at the Stanford Internet Encyclopedia (<http://plato.stanford.edu/entries/qm-bohm>), an article I heartily recommend. My goal with the current paper is both to flesh out the validity of part one of Bell’s two-part argument for nonlocality – that is, the validity of EPR – and also to shed some new light on the similarities between the two parts, which I think the reformulation of EPR in Section III does. For more on Bell’s two-part argument, see also D. Dürr, S. Goldstein, and N. Zanghi, “Quantum Equilibrium and the Role of Operators as Observables in Quantum Theory”, *Journal of Statistical Physics*, 116, 959-1055 (2004), quant-ph/0308038, and also F. Laudisa, “The EPR Argument in a Relational Interpretation of Quantum Mechanics”, *Foundations of Physics Letters*, 14 (2) 2001, 119-132.
- ³¹ While editing this essay, I noticed that this sentence of Bell’s can be read in two different ways, both of which are relevant and important. My initial reading – and the

one which makes this quote fit properly into my discussion – was as follows: the failure of local causality (considered as the conclusion of Bell’s theorem) “cannot be attributed to the ‘incompleteness’ of that theory” – meaning, I thought, that we can’t blame the assumption of incompleteness (which is after all a presupposition of Bell’s derivation) for this result. That is, I thought Bell was here saying: one can’t validly claim that “it’s only if we assume QM isn’t complete that we arrive at a conclusion of nonlocality” since, as EPR showed, if we assume QM *is* complete we also are faced with nonlocality. But I now think (despite the plausibility of the above reading) that Bell intended something slightly different – in effect, the same argument in reverse: the failure of local causality (seen as an obvious aspect of standard quantum theory – in particular the collapse postulate, according to which the wave function associated with a system can change due to a measurement performed at a distant location) “cannot be attributed to the ‘incompleteness’ of the that theory [i.e., orthodox quantum theory].” On this reading, Bell’s sentence is addressed to those who casually dismiss the (allegedly only apparent) nonlocality associated with the collapse of the wave-function by claiming the collapse merely represents an updating of knowledge. But this casual dismissal – according to which the same distant physical state is described by two distinct wave functions, one pre- and one post-collapse – represents a tacit admission that the wave function is (or at least was) not a complete description of the physical state of that distant system. That is, the “epistemic” interpretation of the collapse postulate concedes (unwittingly in most cases) that quantum mechanics is incomplete. I now believe this is the argument Bell actu-

ally had in mind when writing this important sentence. Of course, the conclusion of both readings is the same: nonlocality is a feature of any world described accurately by the quantum predictions, and not merely a feature of a world described by hidden-variables theories. But the routes to this conclusion are different in the two readings. The first reading follows the logic I outline in the main text: EPR says that Completeness $\rightarrow \neg$ Locality while Bell’s Theorem says that Incompleteness $\rightarrow \neg$ Locality, so we must conclude that Locality fails. The second reading follows a logically equivalent but, I think, clarifyingly different structure: EPR says that Locality $\rightarrow \neg$ Completeness while Bell’s theorem shows that \neg Completeness $\rightarrow \neg$ Locality from which it follows that Locality $\rightarrow \neg$ Locality. This last initially appears troubling, but it isn’t: if a certain premise entails its own negation, that premise must be false – for otherwise we could validly derive a contradiction. So we can infer the same conclusion as before: \neg Locality. But this second reading, while arriving at the same conclusion, has a different natural “running commentary”: if you wish to maintain consistency between QM and the principle of local causality, you *must* introduce local hidden-variables (so argued EPR); but you then ultimately find that this project cannot succeed. You *cannot* reproduce all the (empirically correct, it seems) predictions of quantum theory with a local hidden-variables theory. The only way to save Locality forces you into a denial of Locality (assuming you require agreement with the QM predictions). That is, Locality simply cannot be saved – whether or not one regards the QM description of reality as complete.

³² Bell, *op cit*, page 149