As we saw in our readings (Boccio - Polarization), the electric field vector $\vec{E}$ of plane electromagnetic waves lies in a plane perpendicular to the direction of propagation of the wave. If we choose the z-axis as the direction of propagation, we can represent the electric field vector as a 2-dimensional vector in the x-y plane. This means that we will only require two numbers to describe the electric field. Since the polarization state of the light is directly related to the electric field vector, this means that we can also represent the polarization states of the photons by 2-component column vectors or ket vectors of the form

$$|\psi\rangle = \begin{pmatrix}\psi_x \\ \psi_y\end{pmatrix} \quad \text{where we assume the normalization condition} \quad \langle\psi|\psi\rangle = 1 \qquad (01)$$

Examples

$$|x\rangle = \begin{pmatrix}1 \\ 0\end{pmatrix} \rightarrow x-\textit{polarized photon}(\textit{linear or plane polarization})$$

$$|y\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix} \rightarrow y-\textit{polarized photon}(\textit{linear or plane polarization})$$

$$|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ i\end{pmatrix} \rightarrow \textit{Right circular}-\textit{polarized photon} \qquad (02)$$

$$|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ -i\end{pmatrix} \rightarrow \textit{Left circular}-\textit{polarized photon}$$

$$|45\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ 1\end{pmatrix} \rightarrow \textit{photon polarized at } 45^\circ \text{ to the x-axis}$$

$$(\textit{linear or plane polarization})$$

We also have

$$\langle x|x\rangle = 1 = \langle y|y\rangle \text{ and } \langle x|y\rangle = 0 = \langle y|x\rangle \rightarrow \text{orthonormal set}$$

$$\langle R|R\rangle = 1 = \langle L|L\rangle \text{ and } \langle R|L\rangle = 0 = \langle L|R\rangle \rightarrow \text{orthonormal set} \qquad (03)$$

Each of these two sets is a basis for the 2-dimensional vector space of polarization states since any other state vector can be written as a linear combination of them, i.e.,

$$|\psi\rangle = \begin{pmatrix}\psi_x \\ \psi_y\end{pmatrix} = \psi_x\begin{pmatrix}1 \\ 0\end{pmatrix} + \psi_y\begin{pmatrix}0 \\ 1\end{pmatrix} = \psi_x|x\rangle + \psi_y|y\rangle$$

$$|\psi\rangle = \begin{pmatrix}\psi_x \\ \psi_y\end{pmatrix} = \frac{\psi_x - i\psi_y}{2}\begin{pmatrix}1 \\ i\end{pmatrix} + \frac{\psi_x + i\psi_y}{2}\begin{pmatrix}1 \\ -i\end{pmatrix} = \frac{\psi_x - i\psi_y}{\sqrt{2}}|R\rangle + \frac{\psi_x + i\psi_y}{\sqrt{2}}|L\rangle \qquad (04)$$

We can find the components along the basis vectors using

$$\langle x|\psi\rangle = \langle x|(\psi_x|x\rangle + \psi_y|y\rangle) = \psi_x\langle x|x\rangle + \psi_y\langle x|y\rangle = \psi_x$$

$$\langle y|\psi\rangle = \langle y|(\psi_x|x\rangle + \psi_y|y\rangle) = \psi_x\langle y|x\rangle + \psi_y\langle y|y\rangle = \psi_y \qquad (05)$$

or

$$|\psi\rangle = |x\rangle\langle x|\psi\rangle + |y\rangle\langle y|\psi\rangle \tag{06}$$

and similarly

$$|\psi\rangle = |R\rangle\langle R|\psi\rangle + |L\rangle\langle L|\psi\rangle \tag{07}$$

Basically, we are illustrating examples of a **superposition principle**, which says that any arbitrary polarization state can be written as a superposition (linear combination) of x- and y-polarization states or equivalently, as a superposition of right- and left-circularly polarized states.

The action of a polarizer can be consider as a measurement. What are the operators representing such measurements? Clearly, the operators for x- and y-polarizers are given by

$$\hat{O}_x = |x\rangle\langle x| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad \hat{O}_y = |y\rangle\langle y| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{08}$$

If light is polarized at an angle θ from the x-axis, it is in the state

$$|\theta\rangle = \cos\theta|x\rangle + \sin\theta|y\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{09}$$

The operator representing the polarizer at angle θ is (in the x-y basis)

$$\hat{O}_\theta = |\theta\rangle\langle\theta| = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \tag{10}$$

How Does the Polarization State Vector Change in Physical Systems?

Up to now we have been considering devices such as polaroids, which are **"go-nogo"** devices. Some photons get through and some do not for these devices depending on their polarization state.

We now consider devices where all the photons get through no matter what their polarization state is, but the device changes the incident polarization state in some way.

In particular, we consider the example of a **"birefringent"** crystal, such as calcite. A calcite crystal has a **preferred** direction called the **optic** axis. The crystal has a different index of refraction for light polarized parallel to the optic axis than it has for light polarized perpendicular to the optic axis. We assume that the optic axis is in the x-y plane and send a beam of photons in the z-direction. Photons polarized perpendicular to the optic axis are called **ordinary** and are in the state $|o\rangle$ and photons polarized parallel to the optic axis are called **extraordinary** and are in the state $|e\rangle$.

The set of states $\{|o\rangle,|e\rangle\}$ forms an orthonormal basis and general photon states interacting with a calcite crystal are written as superpositions of these basis states.

This is an example of a **general rule in quantum mechanics**.

If we are doing an experiment using a particular measuring device that measures the observable $\hat{Q}$, then we should use as the basis for all states, the eigenvectors of $\hat{Q}$. As we shall see, this requirement pushes us to ask the correct experimental questions (those that quantum mechanics can answer). This particular basis is called the **home** space for the experiment.

Now, as we saw earlier, the phase of a light wave with wavelength λ as it propagates through a medium in the z-direction is given by the quantity

$$\phi = e^{ikz}$$

with (11)

$$k = \frac{2\pi}{\lambda} = \frac{n\omega}{c}$$

where n = index of refraction, $\omega = 2\pi\nu$, ν = frequency and c = speed of light.

Since the phase depends on the index of refraction, the effect of passing through a calcite crystal is to change the r**elative phase** of the $|o\rangle$ and $|e\rangle$ components making up the superposition.

We assume that the state of the photon entering the calcite crystal is

$$|\psi_{in}\rangle = |e\rangle\langle e|\psi_{in}\rangle + |o\rangle\langle o|\psi_{in}\rangle \tag{12}$$

The two components have different indices of refraction n_e and n_o, respectively .

If the beam passes through a length ℓ of calcite, then the state upon leaving is given by inserting phase changes for each component and remembering that the component phases change differently.

$$|\psi_{out}\rangle = e^{ik_e\ell}|e\rangle\langle e|\psi_{in}\rangle + e^{ik_o\ell}|o\rangle\langle o|\psi_{in}\rangle = \hat{U}_\ell|\psi_{in}\rangle \tag{13}$$

where

$$\hat{U}_z = e^{ik_e z}|e\rangle\langle e| + e^{ik_o z}|o\rangle\langle o| \tag{14}$$

is a **"time development"** operator of some sort since ℓ = distance traveled in a time t.

We can represent an arbitrary photon polarization state in the x-y basis as

$$|\psi\rangle = a|x\rangle + be^{i\delta}|y\rangle = \begin{pmatrix} a \\ be^{i\delta} \end{pmatrix} \tag{15}$$

where a, b, and δ. δ is called the **relative phase** between the components. We also have

$$\langle\psi|\psi\rangle = a^2 + b^2 = 1 \tag{16}$$

As we saw above, linear polarization in direction θ (with respect to x-direction)results when $\delta = 0$ and $a = \cos\theta$ and $b = \sin\theta$

$$|\theta\rangle = \cos\theta|x\rangle + \sin\theta|y\rangle = \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{17}$$

so that x-polarization corresponds to $\theta = 0$ and y-polarization corresponds to $\theta = \pi/2$.

Left-circular polarization results when $\delta = -\pi/2$ and $a = b$

$$|L\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + e^{-i\pi/2}|y\rangle\right) = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ -i\end{pmatrix} \tag{18}$$

Right-circular polarization results when $\delta = \pi/2$ and $a = b$

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + e^{i\pi/2}|y\rangle\right) = \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ i\end{pmatrix} \tag{19}$$

All of these polarization states can be generated and manipulated with the use of optical components such as polarizers and retarders constructed using optically active materials as we saw in the extra problems.

Examples

$$\hat{O}_x = |x\rangle\langle x| = \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}$$ = x-polarizer

$$\hat{O}_y = |y\rangle\langle y| = \begin{pmatrix}0 & 0 \\ 0 & 1\end{pmatrix}$$ = y-polarizer

$$\hat{O}_\theta = |\theta\rangle\langle\theta| = \begin{pmatrix}\cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta\end{pmatrix} \tag{20}$$ = polarizer at angle θ

$$\hat{O}_{\lambda/4} = \begin{pmatrix}1 & 0 \\ 0 & e^{-i\pi/2}\end{pmatrix}$$ = 1/4 - wave plate (a retarder)

(phase shift of $\pi/2$ rad = $90°$)

$$\hat{O}_{\lambda/4} = \begin{pmatrix}1 & 0 \\ 0 & e^{-i\pi}\end{pmatrix}$$ = 1/2 - wave plate (a retarder)

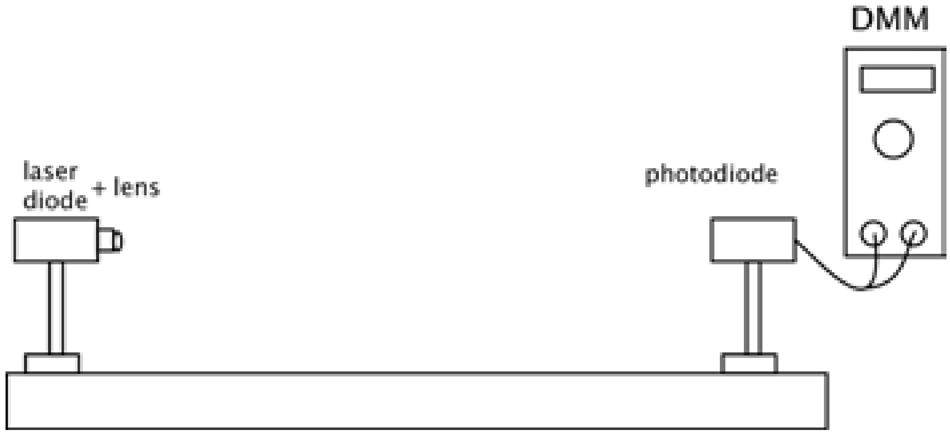
$$\hat{O}_C = \begin{pmatrix}1 & 0 \\ 0 & e^{i\alpha}\end{pmatrix}$$ = birefringent crystal (calcite)

where α depends on path length in the crystal

These last three operators are in the $\{|o\rangle, |e\rangle\}$ basis.

Therefore, each optical component changes the polarization state by rearranging the component magnitudes (although vector length remains = 1) and/or changing the relative phase of the components.
We consider the experimental set-up described below. The figure below

shows the general experimental layout of the light source and detector.



The light source is a laser diode in a holder with an attached lens. It produces light at a wavelength of 635 nm. The (laser-diode + lens + polarizer) system as shown below produces a polarized beam of roughly constant diameter and uniform intensity.

Rotate the polarizer to align the diode polarization(vertical) with the polarizer, as indicated by maximum intensity. The other holders that you will be using are then set up between the first polarizer and the photodiode.

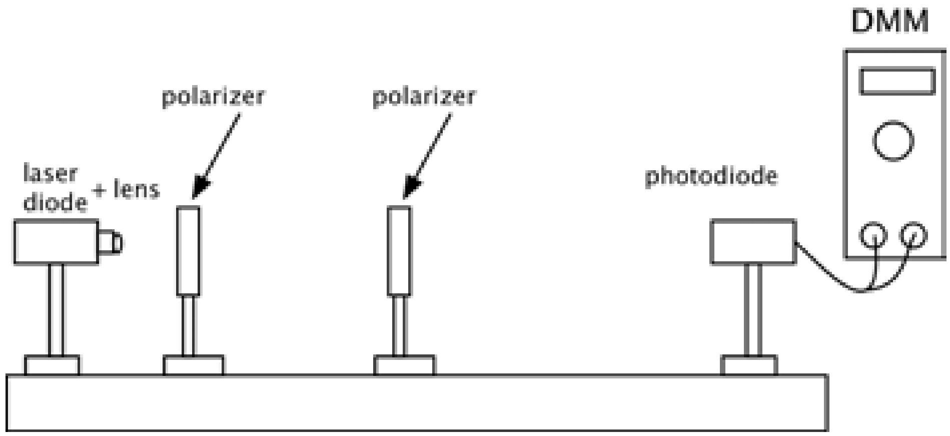
The light detector is a photodiode. It is a semiconductor device which is connected in series with a battery and a large resistor. When light falls on the semiconductor it creates charge carriers in the material, which then flow through the external circuit. By measuring the voltage drop across the series resistor (with a Digital MultiMeter), we can determine the current and infer the amount of light incident on the detector. As with most detectors in the optical frequency range, the photodiode cannot respond to the time variation or phase of the input signal (individual photons), but only the intensity (average number). The current output is therefore proportional to the light intensity, which is given by absolute square of the final quantum amplitude (see calculations below). We will simply take the measured voltage as an estimate of the light intensity (this ignores a proportionality factor which is constant throughout the experiments).

The DMM is connected to the photodiode output and set to read voltage.

Please handle all the optics by the edges. Do not put optical elements on top of each other or slide them across the bench, as they scratch easily.

Part 1 - The first exercise is to measure the transmission of the polarized beam through a linear polarizer as it is rotated about the propagation direction (the beam axis).

We assume that the light exiting the double holder set-up, which is linearly polarized, is polarized in the x-direction (vertically up). The y-direction is then out of the paper (or pointing to the right as you look down the beam axis toward the photodiode). The set-up is shown in the figure below.



We expect a maximum transmission when the polarizer axis is aligned with the polarization of the beam (x-polarization). For a more quantitative analysis(similar to that in the extra problems), we assume that the input beam is linearly polarized in the x-direction so that the incident state is $|x\rangle$. The polarizer axis can be set at any chosen angle θ (as measured clockwise from the x-direction, that is, the vertical). The operator representing the θ-oriented polarizer is given by

$$\hat{O}_\theta = |\theta\rangle\langle\theta| = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \tag{21}$$

Therefore, the final state (after the θ-oriented polarizer) is given by

$$|\,final\rangle = \hat{O}_\theta|x\rangle = |\theta\rangle\langle\theta|x\rangle = |\theta\rangle\big(\cos\theta\langle x|x\rangle + \sin\theta\langle y|x\rangle\big) = \cos\theta|\theta\rangle$$

$$or \tag{22}$$

$$|\,final\rangle = \hat{O}_\theta|x\rangle = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{pmatrix} = \cos\theta\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \cos\theta|\theta\rangle$$

Therefore, the final intensity is proportional to

$$I_0\langle\,final|\,final\rangle = I_0\cos^2\theta\langle\theta|\theta\rangle = I_0\cos^2\theta \tag{23}$$

where I_0 is the intensity without the θ-oriented polarizer present.

This expression needs to be multiplied by an additional factor T_p to account for the extra absorption due to the thickness of the polarizer.

$$\text{output intensity} = I(\theta) = I_0 T_p \cos^2\theta \qquad (24)$$

As expected, the transmitted intensity is maximum when the polarizer is aligned with the input polarization ($\theta = 0$) and minimum when at right angles to the input.

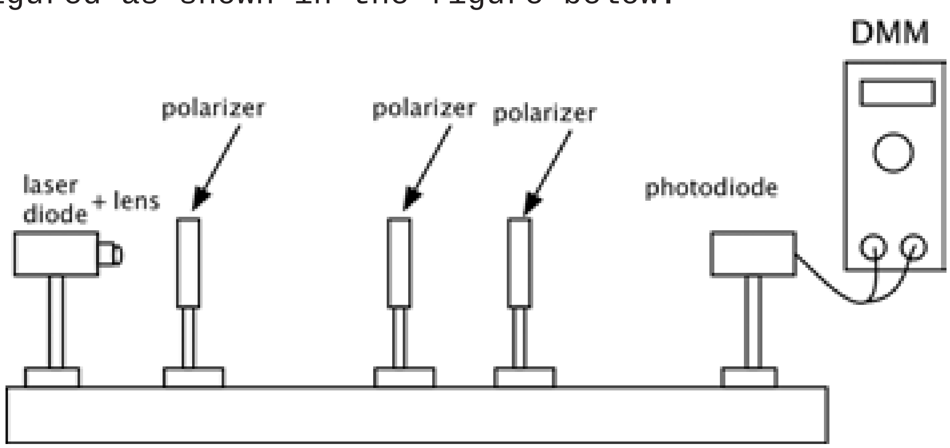
To check this relationship, put one of the rotatable polarizers in a holder and position it in the beam as in the above figure. Measure the transmitted intensity as a function of rotation angle. You can make a plot of the intensity versus angle, which should follow an expression like

$$I(\theta) = A\cos^2(\theta - \beta) + B \qquad (25)$$

The angular shift β accounts for possible misalignment between the polarizer axis and the input beam orientation, while B is a background due to unpolarized transmission or stray room light incident on the photodiode. B will be small in this experiment since we are using a 633 nm filter on the photodiode.

With a couple of auxiliary measurements you can now completely characterize the polarizer. Note that I_0 is greater than A. Use the measured values to compute T_p. Note that a significant fraction of the input beam is absorbed.

Part 2 - We have used the quantum theory of photon polarization to derive the above results which are confirmed experimentally, An even more striking demonstration of this can be done with two polarizers configured as shown in the figure below.



The polarizer closest to the photodiode is oriented orthogonal to the input polarization, that is, in y-direction so it is represented by the operator

$$\hat{O}_y = |y\rangle\langle y| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad (26)$$

This means that the transmitted intensity would be zero without the middle polarizer. Check this experimentally.

Let us assume that the first polarizer is now oriented at an angle θ so that it is represented by the operator

$$\hat{O}_\theta = |\theta\rangle\langle\theta| = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \tag{27}$$

The final state of the system is then given by

$$\begin{aligned} |final\rangle &= \hat{O}_y\hat{O}_\theta|input\rangle = \hat{O}_y\hat{O}_\theta|x\rangle \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{pmatrix} = \begin{pmatrix} 0 \\ \sin\theta\cos\theta \end{pmatrix} = \sin\theta\cos\theta|y\rangle \end{aligned} \tag{28}$$

As expected the final beam is polarized in the y-direction.

We can also derive this result in this manner

$$\begin{aligned} |final\rangle &= \hat{O}_y\hat{O}_\theta|input\rangle = \hat{O}_y\hat{O}_\theta|x\rangle \\ &= (|y\rangle\langle y|)(|\theta\rangle\langle\theta|)|x\rangle = |y\rangle\langle y|\theta\rangle\langle\theta|x\rangle = \sin\theta\cos\theta|y\rangle \end{aligned} \tag{29}$$

Therefore the transmitted intensity is proportional to

$$\langle final|final\rangle = \sin^2\theta\cos^2\theta\langle y|y\rangle = \sin^2\theta\cos^2\theta \tag{30}$$

or including all the absorption factors we have

$$\text{output intensity} = I(\theta) = I_0T_p^2\sin^2\theta\cos^2\theta = \frac{I_0T_p^2}{8}(1-\cos 4\theta) \tag{31}$$

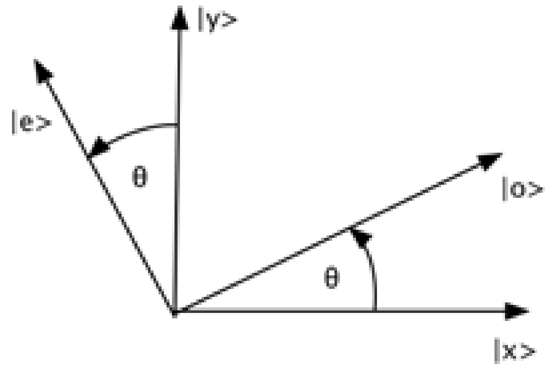
Set up the apparatus by installing the second polarizer and rotating it to get minimum transmission (without the first polarizer). Then install the first polarizer and measure the transmitted intensity as you rotate it. Plot your data and fit it to the output intensity in eq (31) after adding a background term and allowing for a shift in angle as before

$$I(\theta) = A(1-\cos 4(\theta-\phi)) + C \tag{32}$$

Can you explain why the maximum transmission occurs at $45°$?

Part 3 - If a $\lambda/4$ retarder is illuminated with polarized light, the emerging beam will usually be elliptically polarized ($a \neq b$). Circular polarization(pure Left or Right) is a special case occurring when the optic axis is at $\pm 45°$ to the input polarization and the retardation is exactly 1/4 wavelength. Our light source operates at 635 nm. The retarder we will use in this part of the experiment is a 15^{th}-order-$\lambda/4$ plate for 633 nm light.

As usual we assume the input beam is x-polarized. We then suppose that the optical axis makes an angle θ with respect to the x-direction. We then have the orientations shown below



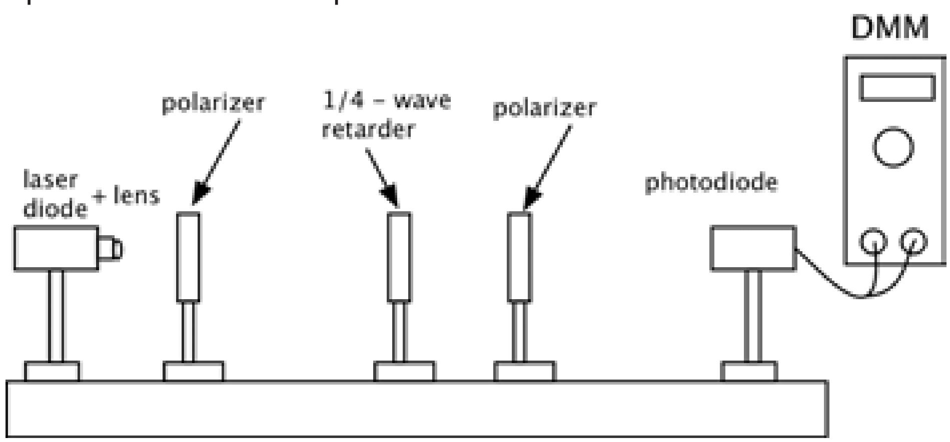
so that

$$|o\rangle = \cos\theta|x\rangle + \sin\theta|y\rangle \; and \; |e\rangle = -\sin\theta|x\rangle + \cos\theta|y\rangle \tag{33}$$

or

$$|x\rangle = \cos\theta|o\rangle - \sin\theta|e\rangle \; and \; |y\rangle = \sin\theta|o\rangle + \cos\theta|e\rangle \tag{34}$$

The experimental set-up is shown below



We then have the quantitative analysis shown below:

$$|input\rangle = |x\rangle = \cos\theta|o\rangle - \sin\theta|e\rangle$$

$$|after\ retarder\rangle = \hat{O}_\gamma|input\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\gamma} \end{pmatrix}\left(\cos\theta\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \sin\theta\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$= \cos\theta\begin{pmatrix} 1 \\ 0 \end{pmatrix} - e^{-i\gamma}\sin\theta\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \cos\theta|o\rangle - e^{-i\gamma}\sin\theta|e\rangle \tag{35}$$

$$|after\ retarder\rangle = \cos\theta(\cos\theta|x\rangle + \sin\theta|y\rangle) - e^{-i\gamma}\sin\theta(-\sin\theta|x\rangle + \cos\theta|y\rangle)$$

$$= (\cos^2\theta + e^{-i\gamma}\sin^2\theta)|x\rangle + \cos\theta\sin\theta(1 - e^{-i\gamma})|y\rangle$$

where we have removed an overall phase factor so that the relative phase change appears only on the $|e\rangle$ component.

Then, since the final polarizer is oriented in the y-direction, we have

$$
\begin{aligned}
|\,final\rangle &= |y\rangle\langle y|after\ retarder\rangle \\
&= |y\rangle\left(\cos^2\theta + e^{-i\gamma}\sin^2\theta\right)\langle y|x\rangle + |y\rangle\cos\theta\sin\theta\left(1-e^{-i\gamma}\right)\langle y|y\rangle \\
&= \cos\theta\sin\theta\left(1-e^{-i\gamma}\right)|y\rangle
\end{aligned} \qquad (36)
$$

so that the transmitted intensity is proportional to

$$
\langle\,final|\,final\rangle = \sin^2\theta\cos^2\theta\left|1-e^{-i\gamma}\right|^2\langle y|y\rangle = \frac{1}{2}(1-\cos\gamma)\sin^2 2\theta \qquad (37)
$$

or

$$
I(\theta) = \frac{I_0 T_r T_p}{2}(1-\cos\gamma)\sin^2 2\theta \qquad (38)
$$

You can check this expression by mounting the polarizer in the beam and orienting it for minimum transmission. Then install the retarder and observe the transmitted intensity as you rotate it. Minimum transmission should occur when the optic axis of the retarder is parallel or perpendicular to the input polarization and maximum transmission should occur at ±45°. Complete the measurement by plotting intensity versus angle and doing a computer fit. Do you get the predicted angular dependence? After measuring T_r and $\cos\gamma$ you can also verify that the intensity is correctly given by the above result.

Although we cannot determine γ by rotating the retarder, it can be found by rotating the polarizer instead. With the polarizer axis perpendicular(y-direction) to the input polarization(x-direction), rotate the retarder to get maximum transmission. That sets the retarder axis at ±45° to the input polarization and gives an output that is close to circular polarization as the retarder can produce with the wavelength used. If the polarizer is set at an angle ϕ with respect to the x-direction, it the gives ϕ-oriented component of the retarder output, so after some algebra the polarizer output is found to be

$$
|input\rangle = |x\rangle = \cos 45^\circ|o\rangle - \sin 45^\circ|e\rangle = \frac{1}{\sqrt{2}}\left(|o\rangle - |e\rangle\right)
$$

$$
\begin{aligned}
|after\ retarder\rangle &= \hat{O}_\gamma|input\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & e^{-i\gamma}\end{pmatrix}\left(\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right) \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} - e^{-i\gamma}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}|o\rangle - e^{-i\gamma}\frac{1}{\sqrt{2}}|e\rangle
\end{aligned} \qquad (39)
$$

$$
\begin{aligned}
|after\ retarder\rangle &= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle\right) - e^{-i\gamma}\frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle\right) \\
&= \frac{1}{2}\left(1+e^{-i\gamma}\right)|x\rangle + \frac{1}{2}\left(1-e^{-i\gamma}\right)|y\rangle = \frac{1}{2}\begin{pmatrix}1+e^{-i\gamma}\\ 1-e^{-i\gamma}\end{pmatrix}
\end{aligned}
$$

Then, since the final polarizer is oriented in the ϕ-direction, we have

$$|\,final\rangle = |\phi\rangle\langle\phi|after\ retarder\rangle = \begin{pmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi \end{pmatrix} \frac{1}{2}\begin{pmatrix} 1+e^{-i\gamma} \\ 1-e^{-i\gamma} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (1+e^{-i\gamma})\cos^2\phi + (1-e^{-i\gamma})\sin\phi\cos\phi \\ (1+e^{-i\gamma})\sin\phi\cos\phi + (1-e^{-i\gamma})\sin^2\phi \end{pmatrix} \tag{40}$$

$$= \frac{1}{2}\begin{pmatrix} \cos\phi(\cos\phi+\sin\phi) + e^{-i\gamma}\cos\phi(\cos\phi-\sin\phi) \\ \sin\phi(\cos\phi+\sin\phi) + e^{-i\gamma}\sin\phi(\cos\phi-\sin\phi) \end{pmatrix}$$

so that the transmitted intensity is proportional to

$$\langle final|\,final\rangle = \frac{1}{4}\begin{pmatrix} \left|\cos\phi(\cos\phi+\sin\phi) + e^{-i\gamma}\cos\phi(\cos\phi-\sin\phi)\right|^2 \\ +\left|\sin\phi(\cos\phi+\sin\phi) + e^{-i\gamma}\sin\phi(\cos\phi-\sin\phi)\right|^2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} \cos^2\phi(\cos\phi+\sin\phi)^2 + \cos^2\phi(\cos\phi-\sin\phi)^2 \\ +2\cos^2\phi(\cos^2\phi-\sin^2\phi)\cos\gamma \end{pmatrix}$$

$$+\frac{1}{4}\begin{pmatrix} \sin^2\phi(\cos\phi+\sin\phi)^2 + \sin^2\phi(\cos\phi-\sin\phi)^2 \\ +2\sin^2\phi(\cos^2\phi-\sin^2\phi)\cos\gamma \end{pmatrix} \tag{41}$$

$$= \frac{1}{4}\left(2\cos^2\phi + 2\sin^2\phi + 2(\cos^2\phi-\sin^2\phi)\cos\gamma\right)$$

$$= \frac{1}{2}\left(1+\cos\gamma\cos 2\phi\right)$$

or

$$I(\phi) = \frac{I_0 T_r T_p}{2}(1+\cos\gamma\cos 2\phi) \tag{42}$$

To complete the characterization of the retarder plate you can now measure the transmission as the polarizer is rotated and fit the data to the result in eq (42) to deduce a value for $\cos\gamma$.

For the retarder we are using the phase of the e and o parts of the beam differ in phase by $15\times 90°$ (in a true quarter-wave plate the phase difference would be $90°$ as in eq (20)).

For 635 nm light we should observe a value of γ given appoximately by

$$\gamma = 90° - (2m+1)*(180°)*\frac{\Delta\lambda}{\lambda} = 90° - (2(15)+1)*(180°)*\frac{2}{633}$$

$$= 90° - 18° = 72°$$

Do you get the expected value of γ?

If you also measure T_r for the retarder alone, then you can verify the overall intensity is correctly by the last two $I(\theta)$ results.

REPORT

Your report should include a discussion of your observations, along with answers to the questions in the text and the required plots.