Physics 6H Freshman Seminar Fall 2004 Assignment #8

Readings:

- Albert - Chapter 1
- Albert - Chapter 2
- Boccio - Postulates
- Boccio - Quantum Details
- Boccio - Superposition B

Summary: This week we will develop the postulates which are the basis for quantum theory, expand on some the details of how quantum theory works and continue looking at superposition.

Everyone Problems:

EP-21 Probabilities
EP-27 Measurement Results
EP-28 Eigenvalue and Eigenvectors
EP-32 Projection Operator Representation
EP-34 Spectral Decomposition

Individual Problems:

EP-29 Orthogonal Basis Vectors
EP-30 Expectation values
EP-31 Bases and Probabilities
EP-33 Operator Algebra
EP-35 Powers
EP-36 Eigenket properties
EP-37 Hardness World
EP-54 Albert Figure 2.8
EP-55 Albert Figure 2.8 redux
EP-67 What comes out?
EP-79 Eigenvalues

Presentations:

- The Postulates
- How It Works
- Position Representation, Wavefunctions
- Time Evolution of Probabilities
- Final Explanation of Albert Experiment
- Electron Interference Experiments
- Stern-Gerlach Experiments

Seminar Break:

Extra Problems:

EP-21. Probabilities - We stated that if a particle is in the state $|\psi\rangle$ and you measure its color, for example, the outcome of the measurement is either $|green\rangle$ with probability $|\langle g|\psi\rangle|^2$ or $|magenta\rangle$ with probability $|\langle m|\psi\rangle|^2$. Let us try this out on a few states.

What are the possible outcomes and with what probability do they occur if the color is measured of a particle in

(a) the state $|hard\rangle$
(b) the state $|soft\rangle$
(c) the state $|\psi\rangle = \sqrt{\frac{3}{4}}|hard\rangle + \frac{1}{2}|soft\rangle$ (use the (hard,soft) basis for the calculation).

EP-27. Measurement Results - Given particles in state $|\alpha\rangle = \frac{1}{\sqrt{83}}\left(-3|1\rangle + 5|2\rangle + 7|3\rangle\right)$ where $\{|1\rangle, |2\rangle, |3\rangle\}$ form an orthonormal basis, what are the possible experimental results for a measurement of $\hat{Y}$ and with what probabilities do they occur is the operator $\hat{Y}$ is (in this basis)

$$\hat{Y} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

EP-28. Eigenvalue and Eigenvectors

Find the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ 5 & 0 & 3 \end{pmatrix}$$

Are the eigenvectors orthogonal? Comment on this.

EP-29. Orthogonal Basis Vectors

Compute the eigenvectors of the matrix operator

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Construct an orthonormal basis set from the eigenvectors of this operator.

EP-30 - Expectation values

Let $R = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$ represent an observable, and $|\Psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$ be an arbitrary state vector(with $|a|^2 + |b|^2 = 1$). Calculate $\langle R^2 \rangle$ in two ways:

(a) Evaluate $\langle R^2 \rangle = \langle \Psi | R^2 | \Psi \rangle$ directly.
(b) Find the eigenvalues and eigenvectors of R, $R|r_n\rangle = r_n|r_n\rangle, n = 1,2$. Expand the state vector as a linear combination of the eigenvectors $|\Psi\rangle = c_1|r_1\rangle + c_2|r_2\rangle$ and evaluate $\langle R^2 \rangle = r_1^2|c_1|^2 + r_2^2|c_2|^2$. Do these results agree with the general results we derived earlier?

EP-31 Bases and Probabilities

The initial state $|\psi_{init}\rangle$ of a quantum system is given in an orthonormal basis consisting of the three states $|\alpha\rangle, |\beta\rangle$ and $|\gamma\rangle$ by the components

$$\langle\alpha|\psi_{init}\rangle = \frac{i}{\sqrt{3}} \quad , \quad \langle\beta|\psi_{init}\rangle = \sqrt{\frac{2}{3}} \quad , \quad \langle\gamma|\psi_{init}\rangle = 0$$

Calculate the probability of finding the system in the state $|\psi_{final}\rangle$ which has the components

$$\langle\alpha|\psi_{final}\rangle = \frac{1+i}{\sqrt{3}} \quad , \quad \langle\beta|\psi_{final}\rangle = \sqrt{\frac{1}{6}} \quad , \quad \langle\gamma|\psi_{final}\rangle = \sqrt{\frac{1}{6}}$$

in the same basis.

EP-32. Projection Operator Representation

Let the states $\{|1\rangle, |2\rangle |3\rangle\}$ form an orthonormal basis. We consider the operator given by $\hat{P}_2 = |2\rangle\langle 2|$. What is the matrix representation of this operator? What are its eigenvalues and eigenvectors. For the arbitrary state $|A\rangle = \frac{1}{\sqrt{83}}\left(-3|1\rangle + 5|2\rangle + 7|3\rangle\right)$, what is the result of $\hat{P}_2|A\rangle$?

EP-33. Operator Algebra

An operator for a two-state system is given by

$$\hat{H} = a\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

where a is a number. Find the eigenvalues and the corresponding eigenkets (linear combinations of $|1\rangle$ and $|2\rangle$, which are eigenkets).

EP-34. Spectral Decomposition

Find the eigenvalues and eigenvectors of the matrix

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Construct the corresponding projection operators, and verify that the matrix can be written in terms of its eigenvalues and eigenvectors. This is the **spectral decomposition** for this matrix.

EP-35. Suppose that we have some operator $\hat{Q}$ such that $\hat{Q}|q\rangle = q|q\rangle$ i.e., $|q\rangle$ is an eigenvector of $\hat{Q}$ with eigenvalue q .

(a) Show that $|q\rangle$ is also an eigenvector of the operators $\hat{Q}^2, \hat{Q}^n$ and $e^{\hat{Q}}$
(b) What are the corresponding eigenvalues?

EP-36. Eigenket properties

Consider a 3-dimensional ket space. If a certain set of orthonormal kets, say $|1\rangle$, $|2\rangle$, and $|3\rangle$, are used as the basis kets, the operators $\hat{A}$ and $\hat{B}$ are represented by

$$\hat{A} \rightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \quad , \quad \hat{B} \rightarrow \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

where a and b are both real numbers.

(a) Obviously $\hat{A}$ exhibits a degenerate spectrum. Does $\hat{B}$ also exhibit a degenerate spectrum?
(b) Show that $\hat{A}$ and $\hat{B}$ commute.
(c) Find a new set of orthonormal kets which are simultaneous eigenkets of both $\hat{A}$ and $\hat{B}$. Specify the eigenkets of $\hat{A}$ and $\hat{B}$. Does your specification of eigenvalues completely characterize each eigenket?

EP-37. Hardness World - Let us define a state using the hardness basis $(|h\rangle,|s\rangle)$

$$|A\rangle = \cos\theta|h\rangle + e^{i\phi}\sin\theta|s\rangle$$

where θ and ϕ are constants.

(a) Is this state normalized? Show your work.
(b) Find the state $|B\rangle$ that is orthogonal to $|A\rangle$. Make sure $|B\rangle$ is normalized.
(c) Express $|h\rangle$ and $|s\rangle$ in the $(|A\rangle,|B\rangle)$ basis.
(d) What are the possible outcomes of a hardness measurement on state $|A\rangle$ and with what probability will each occur?
(e) Express the hardness operator in the $(|A\rangle,|B\rangle)$ basis.

EP-54. Albert Figure 2.8 - Consider the following situations based on Figure 2.8 in Albert:

(a) just as in the figure
(b) a wall in one path (x_3,y_1)
(c) a wall in one path (x_2,y_2)

Make a chart with each row representing one of these situations. In the columns of the chart, give

(1) the state of the particles that emerge at (x_5,y_4)
(2) the probability that a hardness box at (x_5,y_4) measures the hardness to be hard
(3) the probability that a hardness box at (x_5,y_4) measures the hardness to be hard
(4) the probability that a color box at (x_5,y_4) measures the color to be green
(5) the probability that a color box at (x_5,y_4) measures the

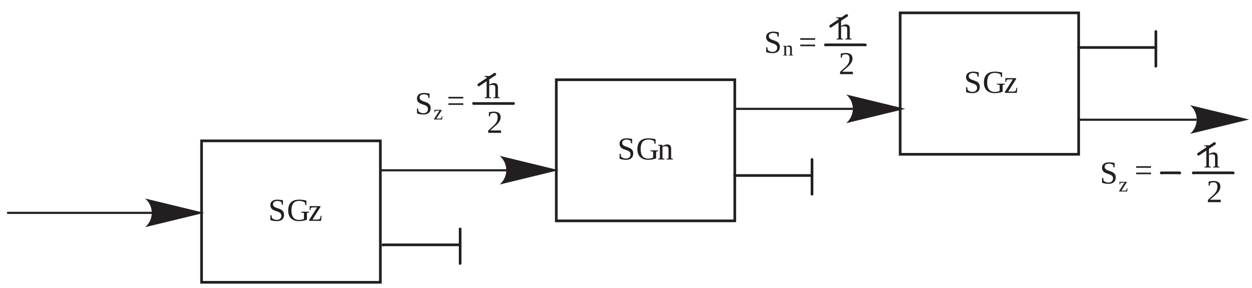
color to be magenta

EP-55. Albert Figure 2.8 redux - Again using Figure 2.8 as a guide, imagine that the harness box is placed in the one path at (x_2,y_2). The hard output of this box is blocked, but the soft output of this box sends the particles along the same path they were on before they entered this additional box.

(a) What is the state of the particles that emerge at (x_5,y_4), what fraction of the particles that enter the apparatus emerge at (x_5,y_4), and what would be the results of measurements of the hardness and color at (x_5,y_4)?
(b) Answer the same questions if instead a color box is placed in the one path at (x_3,y_1) with its magenta output blocked and the green output directing the particles along their original direction.

EP-67. What comes out?

A beam of spin 1/2 particles is sent through series of three Stern-Gerlach measuring devices as shown below:



The first SGz device transmits particles with $\hat{S}_z = \hbar/2$ and filters outs particles with $\hat{S}_z = -\hbar/2$. The second device, an SGn device transmits particles with $\hat{S}_n = \hbar/2$ and filters outs particles with $\hat{S}_n = -\hbar/2$, where the axis $\hat{n}$ makes an angle θ in the x-z plane with respect to the z-axis. Thus the particles passing through this SGn device are in the state $|+\hat{n}\rangle = \cos\frac{\theta}{2}|+\hat{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\hat{z}\rangle$ with the angle $\phi = 0$. A last SGz device transmits particles with $\hat{S}_z = -\hbar/2$ and filters outs particles with $\hat{S}_z = \hbar/2$.

(a) What fraction of the particles transmitted through the first SGz device will survive the third measurement?
(b) How must the angle θ of the SGn device be oriented so as to maximize the number of particles the at are transmitted by the final SGz device? What fraction of the particles survive the third measurement for this value of θ ?
(c) What fraction of the particles survive the last measurement if the SGn device is simply removed from the experiment?

EP-79. Eigenvalues - Determine the eigenvalues and eigenstates of the following matrix

$$\begin{pmatrix} 2 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$