Physics 6H Freshman Seminar Fall 2004 Assignment #10

Readings:

- Boccio Polarization
- Boccio Polaroids, SGs and Slits
- Boccio Photons and Bayes
- Experiment #4

This week we study the two-level photon polarization system as an example of a quantum system. It will illustrate all aspects of quantum theory. In addition, we look at the Stern-Gerlach apparatus and slits.

Everyone Problems:

EP-41 Optical Elements
EP-42 Polaroids
EP-43 Change the Basis
EP-48 Unpolarized Light
EP-52 Quarter-wave plate
EP-58 Time evolution

Individual Problems:

EP-44 Calcite
EP-45 Make a Filter
EP-46 No Change
EP-47 Turpentine
EP-49 What QM is all about
EP-50 More Optical Operators
EP-51 Photons and polarizers
EP-53 What is happening?

Presentations:

- Quantum Theory of Photon Polarization
- Pure States, Unpure States and Density Operators
- Evolution of Polarization States
- Bayesian Thoughts

Experiment #4 Discussion

- Matrix Representations of Optical Elements
- Experiment #1
- Experiment #2
- Experiment #3a
- Experiment #3b

Seminar Break:

Extra Problems:

EP-41. Optical Elements - Suppose three optical elements can be represented by the following three operators in the ($|x\rangle, |y\rangle$) basis.

$$\hat{O}_{\lambda/4} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \qquad \hat{O}_{CP} = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \qquad \hat{O}_{\lambda/2} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(a) What is the probability of a photon in state $|45°\rangle$ getting through the element represented by $\hat{O}_{\lambda/4}$ and what state will it be in if it does?
(b) What is the probability of a photon in state $|0°\rangle$ getting through the element represented by $\hat{O}_{CP}$ and what state will it be in if it does?
(c) What is the probability of a photon in state $|135°\rangle$ getting through the element represented by $\hat{O}_{\lambda/2}$ and what state will it be in if it does?

EP-42. Polaroids - Imagine a situation in which a photon in the $|x\rangle$ state strikes a vertical polaroid. Clearly the probability of the photon getting through the vertical polaroid is 0. Now consider the case of two polaroids with the photon in the $|x\rangle$ state striking a polaroid at $45°$ and then striking a vertical polaroid.

(a) Show that the probability of the photon getting through both polaroids is 1/4.

Consider now the case of three polaroids with the photon in the $|x\rangle$ state striking a polaroid at $30°$ first, then a polaroid at $60°$ and finally a vertical polaroid.

(b) Show that the probability of the photon getting through all three polaroids is 27/64.

EP-43.Change the Basis - In examining light polarization, we have been working in the $(|x\rangle,|y\rangle)$ basis.

(a) Just to show how easy it is to work in other bases, express $|x\rangle$ and $|y\rangle$ in the $(|R\rangle,|L\rangle)$ and $(|45°\rangle,|135°\rangle)$ bases.

(b) If you are working in the $(|R\rangle,|L\rangle)$ basis, what would the operator representing a vertical polaroid look like?

EP-44. Calcite - A photon is polarized at an angle θ to the optic axis is sent in the z direction through a slab of calcite $10^{-2}\ cm$ thick in the z direction. Assume that the optic axis lies in the x-y plane. Calculate, as a function of θ, the transition probability for the photon to emerge left circularly polarized. Sketch the result.

Let the frequency of the light be given by $c/\omega = 5000\,\text{Å}$, and let $n_e = 1.50$ and $n_o = 1.65$ for the calcite.

EP-45. Make a Filter - Using calcite and polaroid, devise a filter that will pass light of frequency $c/\omega = 5000\,\text{Å}$ only if it is right circularly polarized. Use the same calcite parameters as in the previous problem.

EP-46. No Change - What is the condition on the length of a slab of calcite, for frequency ω, such that $|\psi_{out}\rangle$ is always, to within a phase factor, the same as $|\psi_{in}\rangle$?

EP-47. Turpentine - Turpentine is an "optically active" substance. If we send plane polarized light into turpentine then it emerges with its plane of polarization rotated. Specifically, turpentine induces a left-hand rotation of about $5°$ per cm of turpentine that the light traverses. Write down the transition matrix that relates the incident polarization state to the emergent polarization state. Show that this matrix is unitary. Why is that important? Find its eigenvectors and eigenvalues, as a function of the length of turpentine traversed.

EP-48. Unpolarized Light - Unpolarized light traveling in the z direction is sent through a block of calcite whose optic axis lies in the x-y plane. What is the effect of the calcite on the polarization properties of the beam? What will turpentine do to an unpolarized beam ?

EP-49. What QM is all about - Photons polarized at $30°$ to the x-axis are sent through a y-polaroid. An attempt is made to determine how frequently the photons that pass through the polaroid, pass through as "right circularly polarized photons" and how frequently they pass through as "left circularly polarized photons". This attempt is made as follows:

First, a prism that passes only right circularly polarized light is placed between the source of the $30°$ polarized photons and the y-polaroid, and it is determined how frequently the $30°$ photons pass through the y-polaroid. Then this experiment is repeated with a prism that passes only left circularly polarized photons instead of the one that passes only right.

Show by explicit calculation that the sum of the probabilities for passing through the y-polaroid measured in these two experiments is different from the probability that one would measure if there were no prism in the path of the photon and only the y-polaroid.

Relate this experiment to the two-slit diffraction experiment.

EP-50.More Optical Operators - Look again at the three operator in Problem 41. The solution to that problem shows that in part (a) incoming $|45°\rangle$ photons come out as $|L\rangle$ photons with probability 1, that in part (b) incoming $|0°\rangle$ photons were changed to $|L\rangle$ photons with probability 1/2, and in part (c) $|135°\rangle$ photons come out as $|-135°\rangle$ photons with probability 1. But if these operators represent measurements, then we can calculate the probabilities by simply finding modulus-squared of the bra-ket of the output state and the input state. This works for parts (b) and (c), but not for part (a). Therefore, the operator in part (a) must not represent a measurement, which makes sense since it is the only one of the three that is not Hermitian. Below are three more operators.

$$\hat{O}_{45} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad \hat{O}_{\lambda/4} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix} \qquad \hat{O}_{CP} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

Which ones are Hermitian? What is the output state for incoming photons in the $|y\rangle$ state and what is the probability that one of these photons will get through?

EP-51. Photons and polarizers

A photon polarization state for a photon propagating in the z-direction is given by

$$|\psi\rangle = \sqrt{\frac{2}{3}}|x\rangle + \frac{i}{\sqrt{3}}|y\rangle$$

(a) What is the probability that a photon in this state will pass through a polaroid with its transmission axis oriented in the y-direction?

(b) What is the probability that a photon in this state will pass through a polaroid with its transmission axis y' making an angle ϕ with the y-axis ?

(c) A beam carrying N photons per second, each in the state $|\psi\rangle$, is totally absorbed by a black disk with its normal to the surface in the z-direction. How large is the torque exerted on the disk? In which direction does the disk rotate? REMINDER: The photon states $|R\rangle$ and $|L\rangle$ each carry a unit $\hbar$ of angular momentum parallel and antiparallel, respectively, to the direction of propagation of the photon.

EP-52. Quarter-wave plate

A beam of linearly polarized light is incident on a quarter-wave plate (changes relative phase by $90°$) with its direction of polarization oriented at $30°$ to the optic axis. Subsequently, the beam is absorbed by a black disk. Determine the rate angular momentum is transferred to the disk, assuming the beam carries N photons per second.

EP-53. What is happening?

A system of N ideal linear polarizers is arranged in sequence. The transmission axis of the first polarizer makes an angle ϕ/N with the y-axis. The transmission axis of every other polarizer makes an angle ϕ/N with respect to the axis of the preceding polarizer. Thus, the transmission axis of the final polarizer makes an angle ϕ with the y-axis. A beam of y-polarized photons is incident on the first polarizer.

(a) What is the probability that an incident photon is transmitted by the array?
(b) Evaluate the probability of transmission in the limit of large N.
(c) Consider the special case with the angle $\phi = 90°$. Explain why your result is not in conflict with the fact that $\langle x|y\rangle = 0$.

EP-58. Time evolution

The matrix representation of the Hamiltonian for a photon propagating along the optic axis (taken to be the z-axis) of a quartz crystal using the linear polarization states $|x\rangle$ and $|y\rangle$ as a basis is given by

$$\hat{H} = \begin{pmatrix} 0 & -iE_0 \\ iE_0 & 0 \end{pmatrix}$$

(a) What are the eigenstates and eigenvalues of the Hamiltonian?

(b) A photon enters the crystal linearly polarized in the x direction, that is, $|\psi(0)\rangle = |x\rangle$. What is $|\psi(t)\rangle$, the state of the photon at time t ? Express your answer in the $\{|x\rangle, |y\rangle\}$ basis.

(c) What is happening to the polarization of the photon as it travels through the crystal?