Decoherence

Although the principles of quantum mechanics apply in the large-scale world that we inhabit in our daily lives, their consequences seem to be unobservable.

The effects of interference and uncertainty **jitter** are too insignificant in the everyday world to be observed(wavelengths too small).

A new concept of **decoherence**, which is a normal quantum process, says this is not true.

Proponents of decoherence claim that Schrodinger's cat, and indeed all other macroscopic objects, are, in fact quantum objects and must be described by superpositions.

As we saw early this leads to an infinite string of superpositions that we are only able to exit (and see pointers point) by assuming a collapse occurs.

Decoherence advocates, however, claim that the superpositions, which would cause macroscopic objects to have strange properties, effectively disappear in extremely short times and the macroscopic objects become some a new state called a "**mixture**".

They claim that when the decoherence effect is included the Schrodinger cat experiment does not present us with any conceptual difficulties.

Let us remember what the difference between a superposition state and a mixture state is?

Let us imagine we have a box containing a large number of electrons. The box has a hole in it, which allows the electrons to escape one by one, and each such electron that escapes the box is passed through a color box which determines the color of the electron.

If we perform this experiment a large number of times, we obtain the result green half the time and the result magenta half the time.

There are **two different ways**, however, that this result could have come about.

On the one hand, **each electron** in the box might be in a **superposition state**

$$|electron\rangle = \frac{1}{\sqrt{2}}(|green\rangle + |magenta\rangle)$$

so they are neither green nor magenta.

Alternatively, **half the electrons in the box might definitely be green and the other half might definitely be magenta**....they are in a **mixture state**.

How can we determine which of these two alternatives is the case?

We just need to use a **hardness box** instead of a color box.

What happens in each case?

In the first case, if we change from the color to the hardness basis, we have

$$|electron\rangle = \frac{1}{\sqrt{2}}(|green\rangle + |magenta\rangle) = |hard\rangle$$

Thus, we have an eigenstate of hardness!!

This says that every electron will emerge from the hard aperture of the hardness box.

In the case of a mixture,however, where half are green and half are magenta, when we use the hardness box we find half emerge from the hard aperture and half emerge from the soft aperture.

The essential difference is that in a superposition

$$|electron\rangle = c_1|green\rangle + c_2|magenta\rangle$$

we know the probability amplitudes $c_1 \text{ and } c_2$, but in a mixture we only know their absolute values $|c_1| \text{ and } |c_2|$,i.e., in one case we know the entire complex number and in the other case only the magnitude of the complex number.

Let us see how this works(more detailed discussion - go pack to section 12 of notes):

$$|electron - \sup erposition\rangle = c_1|green\rangle + c_2|magenta\rangle$$

In this case there is no way to tell is the electron is green or magenta. Therefore the probability amplitude for an electron in this state to go through some process as a green electron is c_1 and the probability amplitude for an electron in this state to go through some process as a magenta electron is c_2. These two ways are indistinguishable. Thus, the total probability amplitude for this process when the electron is in the superposition state is

$$c_1 + c_2$$

and the total probability is

$$|c_1 + c_2|^2$$

or

$$|electron - mixture\rangle = |c_1||green\rangle + |c_2|e^{i\alpha}|magenta\rangle$$

where α is a completely unknown phase. This state represent a **mixture**.

In this state, the two cases, electron acts like green and electron acts like magenta are distinguishable. Therefore the probability for an electron in this state to go through some process as a green electron is $|c_1|^2$ and the probability for an electron in this state to go through some process as a magenta electron is $|c_2|^2$. Thus, the total probability for this process when the electron is in the mixture state is

$$|c_1|^2 + |c_2|^2$$

Clearly, a superposition allows for possibility of interference and mixtures do not since it is possible for $|c_1 + c_2|^2$ to be zero and $|c_1|^2 + |c_2|^2$ can never be zero (where zero = destructive interference).

The essential element behind the decoherence idea is the recognition that, unless extraordinary precautions are taken in laboratory, **large objects are never isolated from their environments**, i.e., air molecules colliding with it.

Furthermore, their **environments are continually and erratically fluctuating**.

Electrons, atoms, etc, can be studied in isolation - we can **isolate** the effect of their interactions with the environment on measurements, i.e., cool them to absolute zero, put them in a vacuum, etc.

But cats and baseballs are **perpetually buffeted** about by their environment.

As it sits in its box, Schrodinger's cat is ever so slightly shaken by seismic waves from an earthquake in Peru. A tiny wind disturbs the flying baseball's path.

Indeed, in this regard even the perpetual microscopic jittering back and forth of atoms of which large-scale objects are composed can be thought of as part of this continual erratic fluctuation of the environment.

The consequence of these fluctuations according to the adherents of decoherence is that, for all practical purposes, superpositions are always converted into mixtures and quantum behavior (interference effects) vanishes for macroscopic objects.

A further essential element of the notion of decoherence is that it is to some degree an **illusion**.

Let me explain.

The total system consists of both the cat and its environment - **QM says this total system must be in a superposition state**.

But, when we think about cat, we are thinking of only part of total system that cannot be isolated from its environment, and it is that separate part that can be described by a mixture.

In this way, the notion of decoherence does not invalidate the general principle that quantum mechanics applies to all things, big as well as small. It just modifies the behavior of the macroscopic parts of the system.

How does the incessant fluctuation in an object's environment convert its state from a superposition into a mixture?

The basic ideas behind the process can be understood with help of a simple illustrative example..... in this example we need to remember few facts about spin states....

Spin states are only **up or down** in a given direction(only two possibilities) and the most general spin state is a superposition of these two possibilities

$$|spin\rangle = \alpha|up\rangle_z + \beta|down\rangle_z$$
$$|\alpha|^2 + |\beta|^2 = 1$$
$$\Pr ob(up) + \Pr ob(down) = 1$$

Remember α and β are complex numbers and have both a real and imaginary parts, i.e.,

$$\alpha = \alpha_r + i\alpha_i$$
$$\beta = \beta_r + i\beta_i$$

Another way to represent these number is the following:

$$\alpha = \alpha_r + i\alpha_i = |\alpha|\left(\cos\theta_\alpha + i\sin\theta_\alpha\right)$$
$$\beta = \beta_r + i\beta_i = |\beta|\left(\cos\theta_\beta + i\sin\theta_\beta\right)$$

It can be shown that **any spin superposition state is always an eigenstate of the spin along some direction**, that is, if we look in the correct direction (depends on α and β) then we will find the spin either up or down in that direction.

If we specialize to spins restricted to xy plane, we can reduce this expression to form

$$|spin\rangle = A(|up\rangle_z + B|down\rangle_z)$$
$$where \quad |A| = \frac{1}{\sqrt{2}} \ , \ |B| = 1$$

The illustration of decoherence involves a **game**, which goes as follows:

I have a machine that emits electrons. They are traveling along the z-axis, and I guarantee that their spins are perpendicular to this z-axis - that is, their spins point along some direction in xy plane.

Your task is to find that direction.

How do you do it?

You are equipped with a spin direction measuring apparatus(a Stern-Gerlach apparatus=SG) that is oriented perpendicular to incoming beam of electrons(it is in the xy plane).

This SG apparatus splits the beam into two beams, one whose spins are up along the directional axis of SG machine, and other whose spins are down along this axis. That is what SG devices do!

Now you hold up your measuring device up to the beam of electrons and orient it along some randomly chosen direction in xy plane. You will find some of electrons traveling along the upper path and some along the lower path.

Now rotate your apparatus to some other randomly chosen axis, making sure that it still lies in xy plane. You will still you find some electrons taking upper path and some the lower path. As you continue rotating your apparatus this way and that, eventually you will find one particular direction with the important property that **every electron passes along upper path**.

It is always possible to do this according to Quantum Mechanics.

With this last step you have won the game.

The direction you have found is the direction of spin of the particular spin state we are sending into apparatus.

It is important to realize that if you continue swinging your apparatus this way and that, you are eventually sure to win - for, as stated, every superposition state corresponds to a spin eigenstate pointing purely along some direction.

In order to find it, all you must do is keep trying.

But there is **another wrinkle** to my game.

A switch is mounted on my machine, which so far has been in the "**off**" position.

Now I flip it to "**on**" - the switch could equally well be labeled

"**isolated from environment**" (= off)

and

"**not isolated from environment**" (= on)

or alternatively,

"**superposition**"

and

"**mixture**"

Once I do so, no matter how you turn your Stern-Gerlach apparatus, you can **never** find a direction for which all the electrons pass along only one of the two possible paths.

More than that; no matter how you orient it, half of electrons turn out to traverse upper path and half the lower path.

This result is incomprehensible if the state of the electrons emitted by my machine is described by a superposition, for a superposition **always** corresponds to an eigenstate of spin along some direction.

Thus, my machine suddenly seems to be violating the rules of quantum mechanics.

It is not, of course.

Rather, it is now converting its output, which used to be in a superposition state, into a mixture state instead.

Here is how my machine works.

Inside is a device that emits electrons.

When the switch is "**off**" these electron spins are invariably oriented along a definite axis in xy plane.

But when the switch is "**on**" the machine does something more.

Now, just prior to emitting each electron, it spins a roulette wheel. The outer rim of this wheel, rather than being marked off in usual fashion, is labeled in degrees. Thus, when wheel's marble comes to rest, it does so having randomly specified a certain angle.

Call that angle ϕ.

The machine now proceeds to rotate the electron spin by angle ϕ away from its initial direction -- and it then fires electron off into space.

Thus, with the switch "**off**" my machine emits electrons whose spins all point in same direction; but with switch "**on**" the spins are random.

Now, it is easy to see why you found equal numbers of electrons traversing the upper and lower paths of your Stern-Gerlach apparatus when my switch was "**on**"

Had you happened to guess the angle ϕ correctly, you would have found all electrons traversing the upper path. But you were equally likely to have guessed $\phi-180$, in which case you would have found all electrons traversing lower path.

Similarly, had you happened to guess 90°, or 270° away from ϕ, you would have found equal numbers of electrons traversing two paths.

And finally, had you happened to choose some angle α away from ϕ , you would have obtained some definite ratio between the upper and lower paths -- but had you chosen complement $180-\alpha$, you would have obtained the opposite ratio.

By symmetry, for a random set of guesses on your part, you would have found a random distribution of traversals of two paths.

Mathematically, there are **two ways** to describe the state of electrons emitted by my machine with switch "**on**"

First, this state could be described by a superposition, but the angle ϕ is randomly reset prior to each emission.

Example:

Consider a double slit setup with one electron in the apparatus at a time. Suppose each electron is in a superposition with fluctuating coefficients. Then each electron generates its own pattern.

The sum of the individual patterns will not be equal to single interference pattern for many electrons. The electron must be "coherent",that is, have a definite initial phase relationship.

Back to the spins....

We know, however, that each choice of ϕ corresponds to an eigenstate along the ϕ direction, so a second way to describe the state is to say that each individual electron is in one of the quantum states

$$|0\rangle, |1\rangle, |2\rangle, ..., |359\rangle$$

where

$|0\rangle$ is the state where spin is oriented up along the ϕ = 0 axis
$|1\rangle$ is the state where spin is oriented up along the ϕ = 1 axis

. ...

each with a probability of 1/360.

Since all we know is the probability to have a certain ϕ value and not the probability amplitude, this state represents a mixture.

Since the two representations are equivalent we have reached following crucial insight:

a superposition with randomly fluctuating coefficients is equivalent to a mixture!!!

This, then, is how decoherence hopes to operate:

by continually and erratically changing the coefficients of a superposition

thus causing the original **superposition state** to become a **mixture state**.

But how does it do so – and what has all this to do with interactions between the system and the environment?

Here is how the roulette wheel works.

It is meant to model the results of the interaction between electrons emitted by the machine and a complex environment.

The **environment**, in this case, is an external magnetic field applied to the electron. Like a real environment, this field is continually and erratically fluctuating.

Physicists know that, because an electron possesses a spin, it has an interaction with this erratically fluctuating magnetic field.

The two terms in superposition, one having spin up and other spin down, have different interactions with environment. In particular, they will have **slightly different energy values when the magnetic field is present**.

Now, it is general principle of quantum mechanics that time-evolution operator depends on energy of state since the energy operator determines the time-development operator.

Therefore, the two terms in the superposition evolve differently in time.

Furthermore, because the magnetic field is fluctuating erratically, so are these time evolutions.

By exploiting this randomly fluctuating energy difference we can make the coefficients in superposition fluctuate, so as to turn it into a mixture.

The electron gun emits particles whose spins initially all point along a particular direction.

The exact answer looks like

$$|spin,t\rangle = \frac{1}{\sqrt{2}} e^{-iEt/\hbar}\left[|up\rangle_z + e^{i\gamma Bt/2}|down\rangle_z\right]$$

where $e^{-i\gamma t}$ is shorthand notation for the quantity

$$e^{-i\gamma t} = \cos\gamma t - i\sin\gamma t$$ (just a complex number)

Thus, the coefficients of the two terms in the superposition are randomly fluctuating and because they are doing so, the superposition gets converted into a mixture.

Why don't we see the change occur? How long does this conversion process take?

An illustrative example will make clear that decoherence is a **spectacularly rapid process for macroscopic objects**.

Consider a body at rest on surface of Earth.

Imagine that it shifts about vertically ever so slightly. Because it has done so, it has moved to a slightly higher or lower elevation, and so it has shifted its gravitational potential energy which depends on height. This will cause factor $Et/\hbar$, to fluctuate.

If the shifting about is random, the resulting fluctuation in this factor will wipe out the quantum superposition.

Some Numbers:

If a mass M shifts its elevation by height H, its gravitational potential energy has changed by MgH, where g is acceleration due to gravity.

Then, after a time

$$t = \frac{\hbar}{MgH}$$

the factors will have altered enough to cause a significant fluctuation.

If mass M is, say 100 kilograms, and H is only of the order of the dimensions of single atom, say $10^{-10}m$, then this fluctuation occurs in extraordinarily short time of 10^{-27} seconds!

Thus, even the smallest of motions will have an extraordinarily rapid effect on a superposition.

That is an elementary model of **decoherence**.

Its essential elements are as follows.

(I) The environment in which any system is embedded is constantly and irregularly fluctuating.

(II) If a system is described by a superposition, different terms in superposition will interact differently with environment. In particular, they will have different interaction energies.

Therefore,

(III) The time evolution of each of the terms in its superposition is constantly and irregularly fluctuating and differently for each term.

and

(IV) The end result is indistinguishable from a mixture state.

So macroscopic objects will not exhibit quantum behavior (that is, interference effects).

We note that the same calculation for an electron says the coefficient fluctuation time is about 10^{7} seconds or about a year .... certainly long enough to do a quantum experiment and have the electron behave as a superposition and show interference effects.

QM certainly does not make it easy for us!!!!

Up to now in our discussions of measurement, we have tacitly assumed that the macroscopic detector behaves in the same way as microscopic objects such as photons and atoms. But the phenomenon of decoherence shows this assumption was unwarranted.

Let us now correct this error.

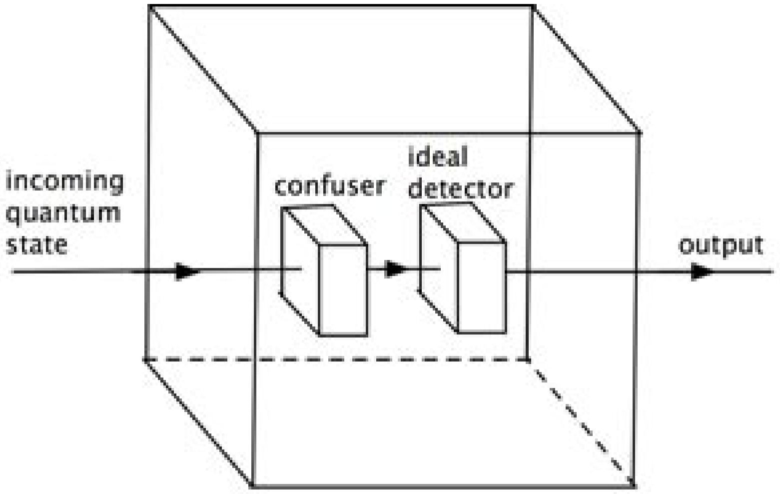
We imagine a macroscopic measuring device as being composed of two parts.

The first part describes all the imperfections of a real measurement process as made by a macroscopic device.

The second part consists of the sort of absolutely perfect device we have been considering up to now.

We will call first part a "**confuser**" and second an "**ideal detector**"

The figure diagrams the configuration we have in mind:



A Macroscopic Detector

In the diagram, the confuser represents all the unwanted interactions of the macroscopic detector with the environment and with its own internal states, which themselves constitute an irregularly fluctuating intrusion into its behavior(it mimics the coupling between a macroscopic detector and its irregularly fluctuating environment).

We have no information about these interactions, which means that they cannot be included in the quantum-mechanical treatment of the detector's behavior.

We can only treat them as unknown and uncontrollable perturbations.

How does such a two-part measuring device work?

Our system enters the device in a certain state. This can be any type of state: an eigenstate, a mixture or a superposition.

In any event, **the state is well-defined**.

But now these systems enter the confuser, and at this point everything changes.

An element of randomness enters our description, arising from process of decoherence. Recall that effect of this randomness is to make a superposition indistinguishable from a mixture and **when the state being measured is a mixture, it turns out that the projection postulate is not required.**

This can be seen as follows:

We prepare a collection of atoms, some in level 2 and some in level 3 of a three level system. Suppose for the sake of argument that 10% are in 2 and 90% in 3.

Such a collection is, of course, simply a mixture as we have already encountered.

We then measure emitted photons(from decays to level 1), allowing sufficient time for entire ensemble of atoms to decay.

Since each individual atom is in a definite level, the result of each measurement is perfectly straightforward, and the projection postulate is not required.

We will not be able to predict in advance which energy each particular photon will have, but this is no cause for concern. We simply do not have sufficient information - even in the classical universe, it is seldom possible to predict an event with this kind of certainty.

But we do know that we will find 10% of the decay photons to be of low energy and 90% of high energy. Results like this lend support to our conventional understanding of macroscopic reality as having an independent existence and well-defined attributes, regardless of the act of measurement.

Notice that, in this case, the initial state of the atom or atoms corresponds directly to one of eigenstates of detection apparatus. As long as this is true, measurement in quantum mechanics does not contradict our naive view of reality.

However, as we have seen time and time again, a special feature of quantum mechanics is that it allows states other than simple eigenstates.

The situation changes dramatically if we consider them as we have seen over an over again.

Now no matter what input state goes into the confuser, its output will effectively be a mixture. We have a loss of quantum interference.

This state now enters the second component of our large-scale measuring device, the perfect detector.

In this situation we again do not need to use the projection postulate.

In this way the phenomenon of decoherence solves measurement problem by eliminating the need for this strange and unsatisfactory postulate or so its adherents claim.

This has been a simplified view of a very complicated mathematical theory of decoherence. The full theory also seems to offer a possible explanation of the mechanism for the collapse process when it is needed.

It is not clear how ever that decoherence produce a perfect mixture state. If it does not then, the quantum entanglement remains and is even more complex.

It is the subject of much dispute at this time.